Portfolio Statistics

Contents

- Expected returns and standard deviation
- Covariance and correlation
- Probabilities and expected returns

Relationship between Mean Return and Expected Return

Mean (Average) Return

Historical return that was actually earned in the past

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Expected Return

The return an investor expects to earn in the future

Given a long enough time period, we can EXPECT that the future expected return will be equal to the average historical return.

Expected Returns

Expected Return is a simple average return (arithmetic mean)

$E(r) = (Rp_1 + Rp_2 + Rp_3 + \ldots + Rp_n)/N$

Assumes all occurrences are equally likely

What if not all occurrences are equally likely?

Weighted Means

Estimates of the return expected from individual assets combined with investors views on risk drive investment decisions.

This involves not only having a "base" case and also estimating the possible deviation from this base case

Weighted Mean: Allows for different weights for different observations

$$E(r) = w_1R_1 + w_2R_2 + ... \; w_nR_n$$

Sum of the weights must equal 1

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Example: Expected Return with Probabilities

Given the two investments below, which investment has the higher expect return?

Case	Security A Expected Return	Security A Probability
Bad Case	-10%	33%
Base Case	15%	33%
Good Case	25%	33%

E(r) =
(-10%*33%)+(15%*33%)+(25%*33%)
=9.9%

Case	Security B Expected Return	Security B Probability
Bad Case	-10%	10%
Base Case	15%	50%
Good Case	25%	40%

E(r) =
(-10%*10%)+(15%*50%)+(25%*40%)
=16.5%

Excel Formula
= SUMPRODUCT(array1,array2,array3,...)

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Covariance

Covariance is a measure of how the returns on two assets move or do not move in same direction.

Similar to variance calculation, but we are comparing two items deviations from their means

$$\mathrm{Cov}(X,Y) = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{(n-1)}$$

if > 0 positive relationship, if < 0 negative relationship

But units make it difficult to interpret the strength of the relationship

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Correlation (aka Correlation Coefficient)

Like covariance, correlations tell you if two assets are positive or inversely related

Unlike covariance, correlation easily tells you the degree of the positive or negative relationship

Easier to interpret as the range of values is scaled and is always -1 to +1

$$\mathrm{Corr}(X,Y) = \rho_{x,y} = \frac{\mathrm{Cov}(X,Y)}{\sigma_x \sigma_y}$$

Equals covariance divided by each standard deviation

Excel Formula:
= CORREL(array1,array2)

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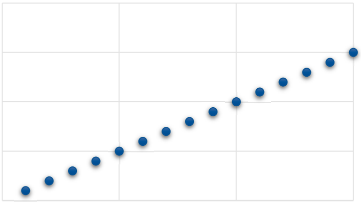
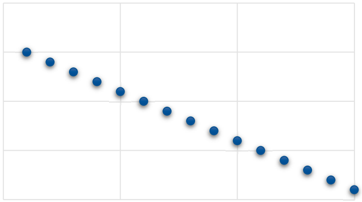
Scatterplots



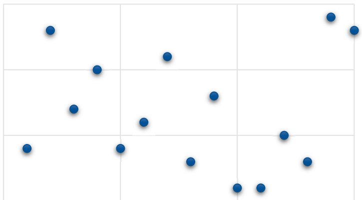


Correlation = -1

Perfect negative correlation and they move in the opposite direction

Correlation = +1

Perfect correlation; Both move proportionally in the same direction



Correlation = zero

No relationship exists; If one moves, no predictions can be made on the movement of the other

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Correlation – Other Key Points

Used to determine what asset to add to a portfolio to increase diversification

The lower the correlation, the more diversification benefits

There are no diversification benefits if the correlation is +1

Correlation does not equal Causation (Spurious)

Snow boots and car accidents have a correlation, but snow boots do not cause car accidents

Correlation between the variables is from their relationship to a third variable – not each other

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Correlation Matrix

	Small Stocks	Large Stocks	LT Corp Bonds	LT Gov Bonds	IT Gov Bonds	Treasury Bills
Small Stocks	1.00					
Large Stocks	0.80	1.00				
LT Corp Bonds	0.04	0.15	1.00			
LT Gov Bonds	-0.10	0.00	0.90	1.00		
IT Gov Bonds	-0.11	-0.03	0.86	0.86	1.00	
Treasury Bills	-0.08	-0.02	0.16	0.18	0.47	1.00

Asset-class Correlations 1926–2015

Source: Morningstar

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Portfolio Expected Returns and Standard Deviation

Expected Return

Very straight forward calculation

Simple weighted average of the investments

$$E(r)_P = E(r)_1 W_1 + E(r)_2 W_2 + \ldots E(r)_n W_n$$

Standard Deviation (Volatility)

More complex and complexity increases with the number of investments

Cannot just take weighted average

$$Stan\ Dev_P = \sigma_p = \sqrt{\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2 w_1 w_2 Corr_{(1,2)} \sigma_1 \sigma_2}$$

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Standard Deviation of a Two Asset Portfolio

Variance of each asset

Multiplied by each of the weightings squared

Correlations: Measures how assets move together

$$Stan\ Dev_P = \sigma_P = \sqrt{\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2w_1 w_2 Corr_{(1,2)}\sigma_1\sigma_2}$$

Easier to compare and interpret than variance

$$Stan\ Dev_P = \sqrt{Variance}$$

$$\sigma_P = \sqrt{\sigma^2}$$

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Diversification Effect

$$Stan\ Dev_P = \sigma_P = \sqrt{\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2w_1 w_2 Corr_{(1,2)}\sigma_1\sigma_2}$$

Perfect Positive Correlation =1	$\sigma_P = \sigma_1 = \sigma_2$	No benefits from diversification
-1< Correlation < +1	Stan Dev of portfolio will be less than weighted avg of assets	Diversification benefits increase as correlation declines
Perfect Negative Correlation = -1	A perfect hedge $\sigma_P = 0$	Stan Dev of the Portfolio is zero: risk free

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Other Key Points

Adding more assets will generally decrease portfolio volatility but have diminishing benefits

Assets can be broaden to industries, asset classes, styles and regions

Adding asset classes that are highly correlated is redundant

Achieves little benefit and adds to costs.

Optimal portfolio reduces risk without sacrificing returns