Modern Portfolio Theory

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What is Modern Portfolio Theory?

Maximize expected return by selecting the weights of various assets

Pioneered by Harry Markowitz in 1952 but is still very relevant

Risk and Return

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Key Assumptions

RETURNS DISTRIBUTION

Returns are distributed along a normal "bell curve"

CORRELATIONS

Correlations do not change over time

RATIONAL AND RISK AVERSE INVESTORS

Investors will make rational investment decisions and will avoid unnecessary risk

RISK

Asset volatility is known and constant

ECONOMIC FRICTION

No taxes or transaction costs

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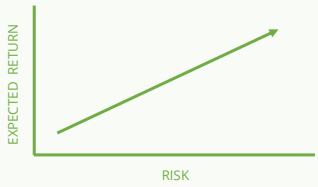
Features

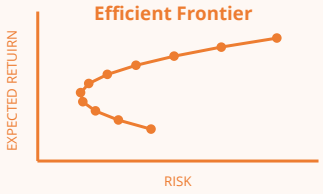


RISK VS. RETURN

Given a level of acceptable risk, an investor should seek out the highest level of return

MEAN VARIANCE OPTIMIZATION

Mean Variance Optimization: process by which investors will allocate assets based on the risk-reward trade-off



EFFICIENT FRONTIER

If all portfolios of varying risk were plotted on a graph, the top edge would represent the most efficient options

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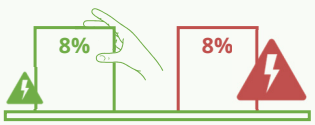
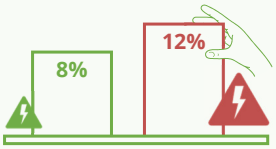
Risk vs. Return

Assumption: All investors are risk averse



Given two portfolios that offer the **same expected return**, investors will **prefer the less risky one**



An investor will **take on increased risk only if compensated** by higher expected returns

Different investors will evaluate how much risk they are willing to take based on their **risk tolerance**

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Example: Risk / Return

Portfolio	Risk (SD)	Return	Trade-off (return/risk)
Portfolio A	10%	15%	1.5
Portfolio B	10%	20%	2
Portfolio C	15%	25%	1.67

MPT assumes **all investors will choose Portfolio B over Portfolio A**

An investor's risk tolerance will determine if the investor prefers Portfolio B or C

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Mean Variance Optimization (MVO)

A framework for determining **how much weight to allocate to each asset in a portfolio** to either maximize the expected return of the portfolio for an expected level of risk, or minimize the expected risk given an expected return

GOAL

Identify the portfolio weights which will deliver the highest return

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MVO Example – Target Portfolio Risk

TARGET
PORTFOLIO RISK
21%

Asset	Expected Return	Risk	Weight Option 1	Weight Option 2	Weight Option 3
Asset A	8%	2%	50%	25%	20%
Asset B	12%	35%	30%	35%	55%
Asset C	15%	5%	20%	40%	25%
Portfolio Risk			21%	21%	21%
Portfolio Return			**10.6%**	**12.2%**	**12.0%**

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MVO Example – Target Portfolio Return

TARGET
PORTFOLIO RETURN
7%

Asset	Expected Return	Risk	Weight Option 1	Weight Option 2	Weight Option 3
Asset A	8%	2%	25%	25%	17%
Asset B	12%	35%	17%	21%	25%
Asset C	15%	5%	20%	17%	17%
Portfolio Return			7%	7%	7%
Portfolio Risk			**13.9%**	**12.0%**	**12.3%**

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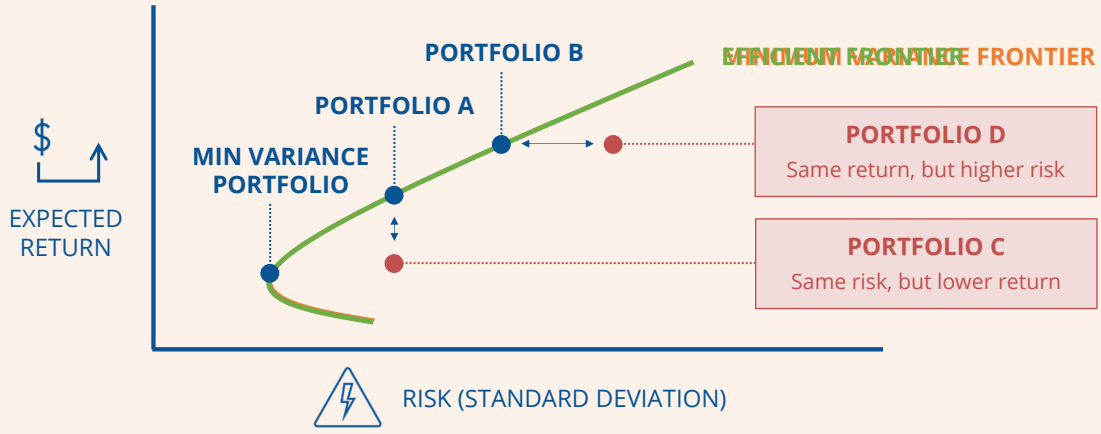
Efficient Frontier



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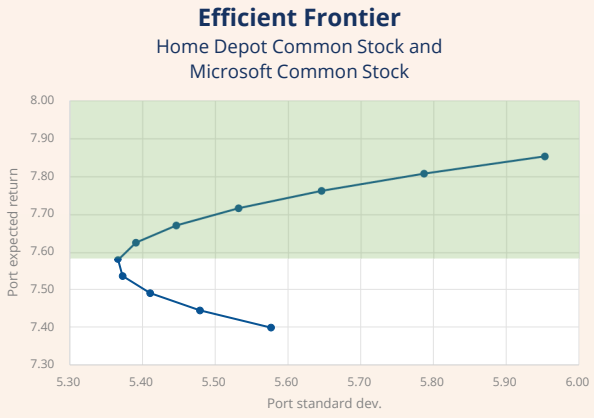
Efficient Frontier Example

	A	B	C	D
39	Expected return on the portfolio			
40		Home Depot Allocation	Standard dev.	Expected return
41			5.39	7.63
42		0.0%	5.95	7.85
43		10.0%	5.79	7.81
44		20.0%	5.65	7.76
45		30.0%	5.53	7.72
46		40.0%	5.45	7.67
47		50.0%	5.39	7.63
48		60.0%	5.37	7.58
49		70.0%	5.37	7.54
50		80.0%	5.41	7.49
51		90.0%	5.48	7.44
52		100.0%	5.58	7.40



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Capital Market Theory

CAPITAL ALLOCATION LINE (CAL)

ALL POTENTIAL PORTFOLIOS

SHARPE RATIO $= \frac{(R_p - R_f)}{\sigma_p}$

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Capital Allocation Line Example

	B	C	D	E
39	Expected return on the portfolio			
40	Home Depot Allocation	Port standard dev.	Port expected return	Sharpe ratio
41		5.39	7.63	1.308
42	0.0%	5.95	7.85	1.223
43	10.0%	5.79	7.81	1.250
44	20.0%	5.65	7.76	1.273
45	30.0%	5.53	7.72	1.291
46	40.0%	5.45	7.67	1.303
47	50.0%	5.39	7.63	1.308
48	60.0%	5.37	7.58	1.306
49	70.0%	5.37	7.54	1.296
50	80.0%	5.41	7.49	1.278
51	90.0%	5.48	7.44	1.254
52	100.0%	5.58	7.40	1.224

	B	C	D
56	Portfolio standard deviation	5.4	
57	Risk free rate	0.574	
58	Portfolio Exp return	7.63	=C17*C29+D17*C30
59	Sharpe ratio	1.308	=(C58-C57)/C56

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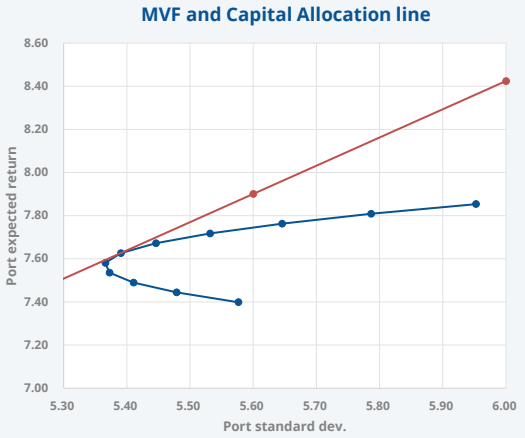
Capital Allocation Line Example

	B	C	D
61	Capital allocation line		
62	Max sharpe ratio	1.3083	
63	Port. Std dev.	Port exp. rtn.	
64	0.00	0.57	=$C$62*B64+$C$25
65	0.40	1.10	=$C$62*B65+$C$25
66	0.80	1.62	=$C$62*B66+$C$25
67	1.20	2.14	=$C$62*B67+$C$25
68	1.60	2.67	=$C$62*B68+$C$25
69	2.00	3.19	=$C$62*B69+$C$25
70	2.40	3.71	=$C$62*B70+$C$25
71	2.80	4.24	=$C$62*B71+$C$25
72	3.20	4.76	=$C$62*B72+$C$25

See supporting Excel file for complete example data



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Criticisms

- TOO SIMPLE
- BACKWARD LOOKING
- ASSUMES ALL INVESTORS ACT RATIONALLY
- ONLY RISKY ASSETS
- ASSUMES MARKETS ARE PERFECTLY EFFICIENT

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Introducing CAPM

WHAT DOES IT DO?

Pricing model that explains the relationship between expected **return and market risk**

HISTORY

Introduced independently by Sharpe, Lintner, Treynor, and Mossin and builds on modern portfolio theory

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Capital Asset Pricing Model (CAPM)

Only market risk should affect asset prices

SYSTEMATIC RISK
Market Risk

MEASURED BY BETA.
This is the part of risk explained by the company's exposure to market factors.

CANNOT BE DIVERSIFIED AWAY!

UNSYSTEMATIC RISK
Company Specific Risk

RISK THAT IS LIMITED TO A PARTICULAR ASSET

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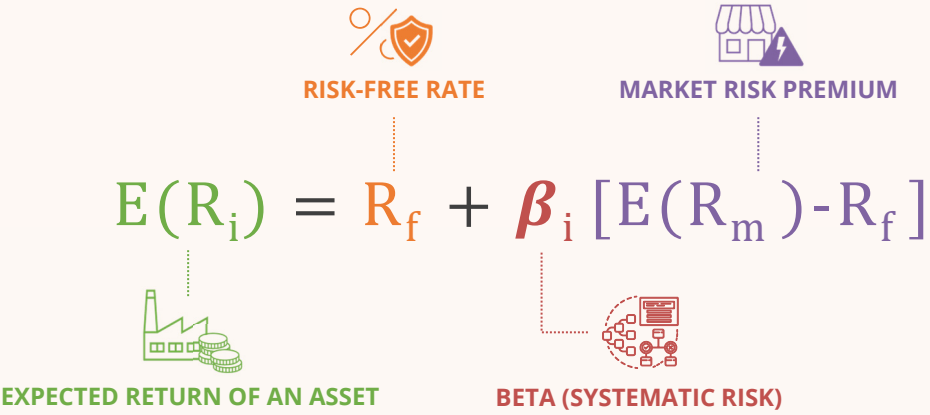
Capital Asset Pricing Model (CAPM)



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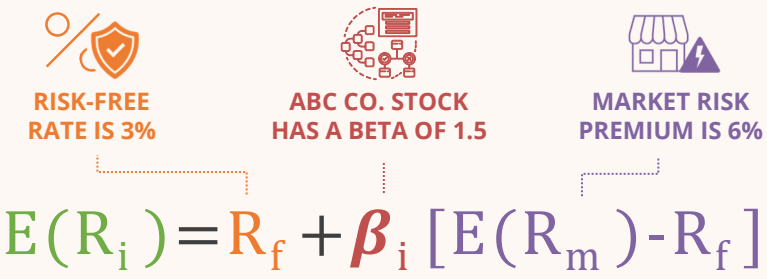
CAPM - Example



Expected Return (ABC) = **3%** + **1.5** x **6%**

Expected Return (ABC) = **12%**

This is **greater than the return on the market** (9%) since the beta is greater than 1

There is **more systematic risk than the market**

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Risk-free Rate

THEORY

Rate of return of an asset that produces a **constant known rate of return** in **all future economic states**

PRACTICAL

Rate of return of an investment with **guaranteed cash flows**

Rate that serves as a **reference rate** for valuing other investments

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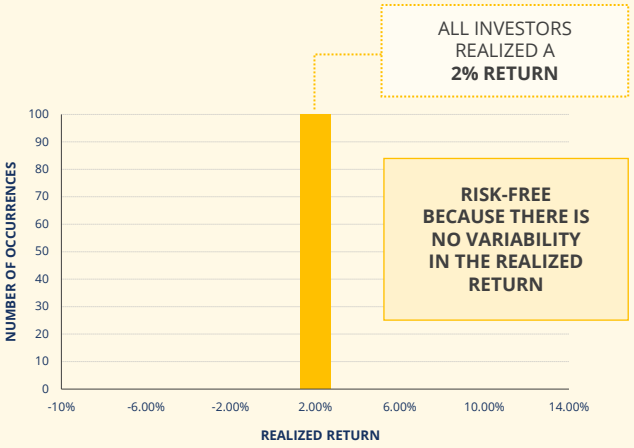
Risk-Free Rate Example

100 INVESTORS

1-YEAR TIME HORIZONS

BUYING A 1-YEAR TREASURY BILL



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CAPM and Beta

TOTAL RISK

STANDARD DEVIATION σ

=

SYSTEMATIC RISK

Beta β

+

UNSYSTEMATIC RISK

EXPOSURE TO MARKET RISKS

SENSITIVITY TO BROAD FACTORS

MEASURES RELATIVE RISK, NOT TOTAL RISK

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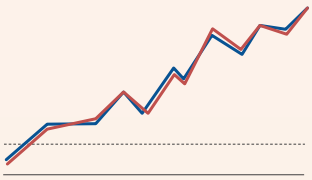
Interpreting Beta

1



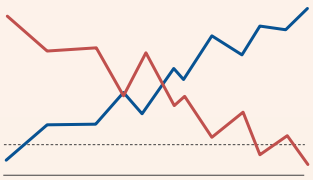
Asset tends to move in the same direction and the same percentage as the market

0

Uncorrelated to the market

No systematic risk

−



The asset tends to move in the opposite direction

Inverse relationship

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Market Risk Premium (MRP)

CAPM MRP
The return demanded for taking equity market-wide risk

→ Premium for investing in the market portfolio, relative to the risk-free rate

$$E(R_i) = R_f + \beta_I [E(R_m) - R_f] =$$ **MARKET RISK PREMIUM**

EXPECTED RETURN FOR THE MARKET – RISK-FREE RATE

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Estimating MRP: Historic Premium Approach

MARKET RISK PREMIUM = **HISTORIC AVERAGE MARKET RETURN** − **HISTORICAL RISK-FREE RATE**

AVERAGES

PERIOD	STOCKS TO 10-YEAR TREASURY US BOND
1928 - 2019	6.43%
1970 - 2019	4.50%
2010 - 2019	9.67%

The MRP is not constant, differing historic time periods and calculation methods will affect its value

Source: Damodaran Online

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Estimating MRP: Forward Looking Estimates Approach

FUTURE ECONOMIC AND FINANCIAL VARIABLES

HISTORICAL MRPS ARE IGNORED

EX-ANTE ESTIMATES

MRP = 

FORWARD DIVIDEND YIELD + 

EARNINGS GROWTH RATE - 

RISK-FREE RATE

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Estimating MRP: Forward Looking Estimates Approaches

MACROECONOMIC MODELS

Based on **identifying relationships** between **economic factors** and **security performance**

SURVEY

Global survey of opinions and views of industry experts

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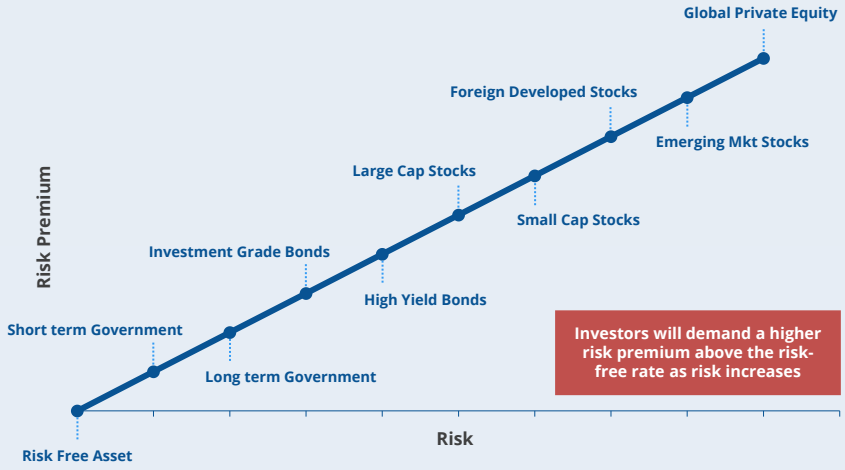
Asset Class Risk Premiums



PRIMARY DETERMINANTS OF RISK PREMIUMS

- Risk Aversion
- Economic Risk
- Inflation and Interest Rates
- Information / Transparency
- Liquidity
- Government Policy

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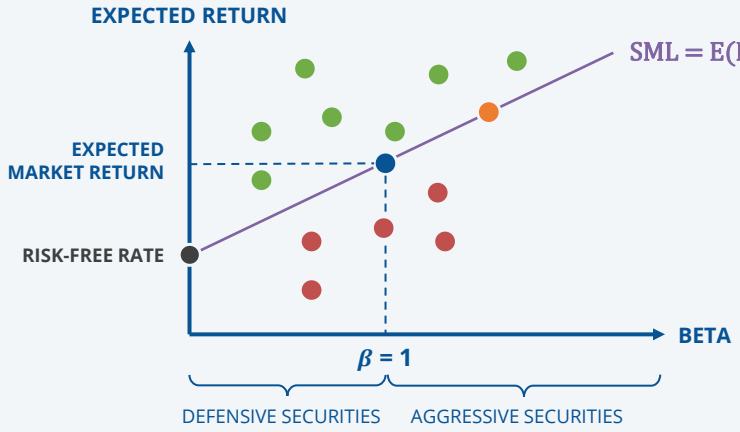
Security Market Line (SML)



$$SML = E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

- UNDERVALUED
- PROPERLY VALUED
- MARKET PORTFOLIO
- OVERVALUED

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A graphical representation of the capital asset pricing model (CAPM)

X-AXIS

Systematic Risk or Beta

Y-AXIS

Expected Return or Required Return

Can be used to determine if a security will provide more (or less) return given its level of risk

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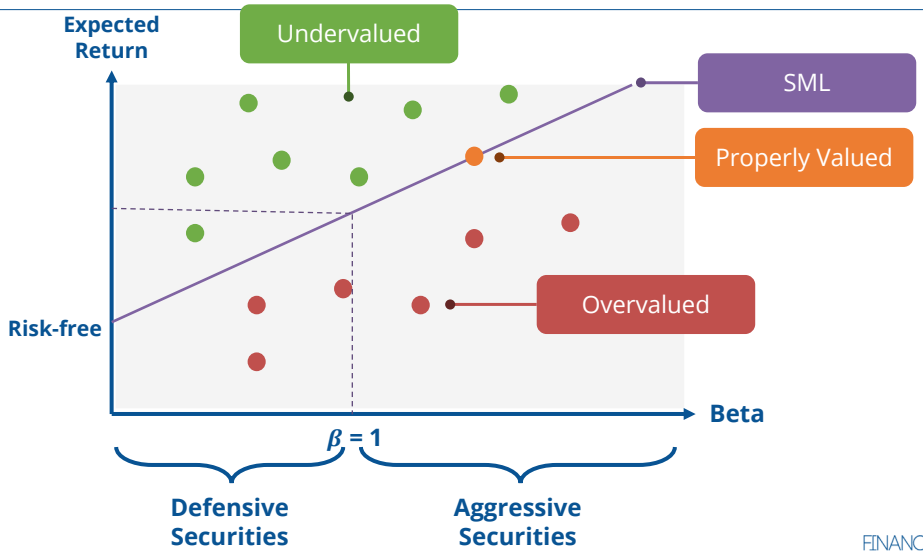
$SML = E(R_i) = R_f + \beta_i[E(R_m) - R_f]$



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Black-Litterman Model – Introduction

A tool for investors to **calculate the optimal portfolio weights under specified parameters** developed to **mitigate problems related to mean-variance optimization** (MVO)

PROBLEMS WITH UNCONSTRAINED MVO RESULTS

Sensitive to expected return inputs

Efficient portfolios highly concentrated in small number of assets

Does not allow for active investment view

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Black-Litterman Model – Introduction

Allows investors to combine their unique forecasts of expected returns ("active views") with passive "equilibrium" expected returns

- ☑ **STABLE**
- ☑ **EFFICIENT**
- ☑ **DIVERSIFIED**

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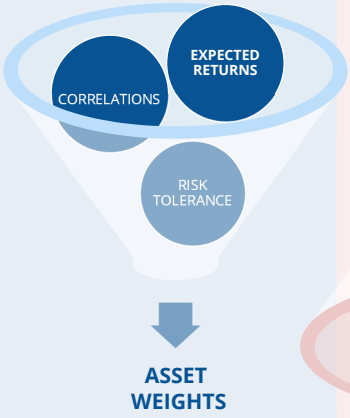
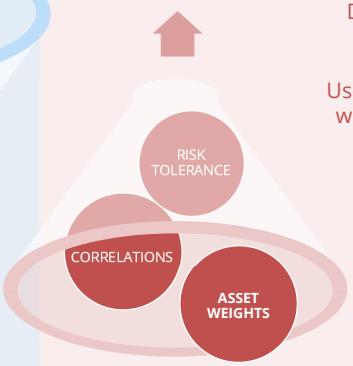
Expected Returns – Reverse Optimization

MEAN VARIANCE OPTIMIZATION



EXPECTED RETURNS



BLACK-LITTERMAN MODEL

Derived using a reverse optimization method

Uses "equilibrium" portfolio weights as a starting point

Returns are the set of returns that clear the market

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Expected Returns – Reverse Optimization

BL ASSET WEIGHTS

Typically based on the market value or **market capitalization** of assets

The **relative fractions** of the value of each firm's stock market capitalization (market weights) against the whole market

Also referred to as **neutral weights**; equal to market portfolio

Returns are backed out through reverse optimization such that they are the returns that would have been used for a mean-variance optimization to get the portfolio weights consistent with the market weights

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Reverse Optimization - Example

SOLVE FOR EXPECTED RETURN OF EACH ASSET"EQUILIBRIUM"

Asset Class	Market Cap	Market Weight	Risk Free	+	Beta	x	MRP	=	Expected Return
A	$70	43.8%	2%		0.39		4%		3.5%
B	$50	31.3%	2%		2.00		4%		10.0%
C	$30	18.8%	2%		0.60		4%		4.4%
D	$10	6.3%	2%		1.50		4%		8.0%
Global Market	**$160**	**100.0%**			**1.00**				**6.0%**

WEIGHT AVERAGE OF ALL BETAS

WEIGHT AVERAGE OF ALL EQUILIBRIUM ASSET RETURNS

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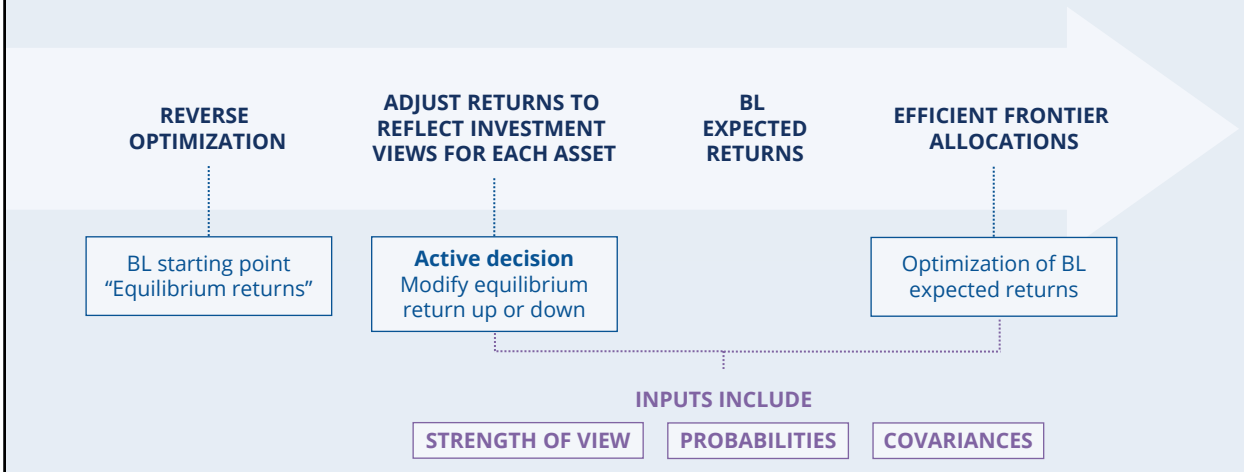
Adding Investment Views



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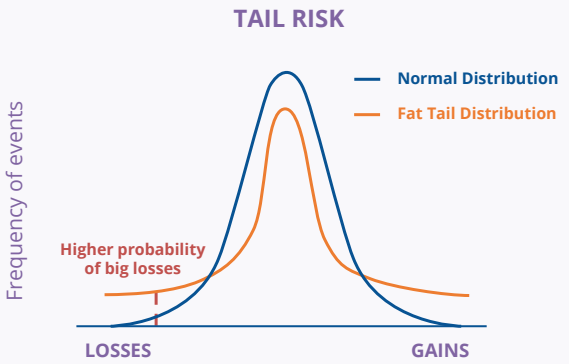
Building Resilient Portfolios



BUILD RESILIENT PORTFOLIOS

Assess and account for uncertainty in MVO assumptions

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What is Robust Optimization?

A statistically driven approach that addresses the difficulty in making estimates with traditional Mean Variance Optimization

Aims to **resolve the high sensitivity to inputs** of the markowitz mean-variance model

Variations and estimation errors of covariance and correlations, and expected returns of assets in various market environments

Stable solutions

Portfolios with a **lower turnover rate** and **suitable for long term planning**

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How Robust Optimization Enhances MVO

MVO USES POINT ESTIMATES FOR RETURNS AND CORRELATIONS

- Focus on simple mean or average estimates
- Like having one foot in freezing water, one foot in boiling water – and on average feeling fine
- Recessions and crisis are binary, not average outcomes

RO USES "UNCERTAINTY SETS"

- Not a point estimate
- Attempts to incorporate all (or most) possible realizations / paths
- Incorporates estimation risk
- Helps maximize possible potential returns under negative scenarios

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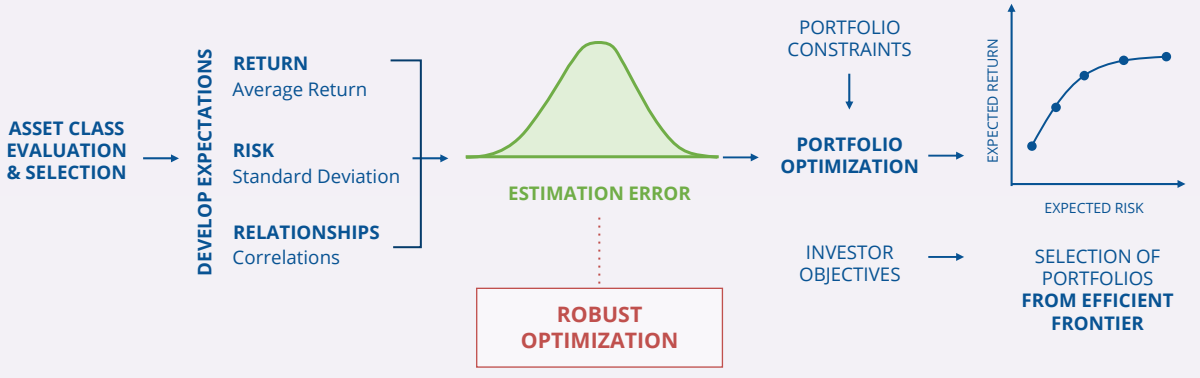
Where does it fit?



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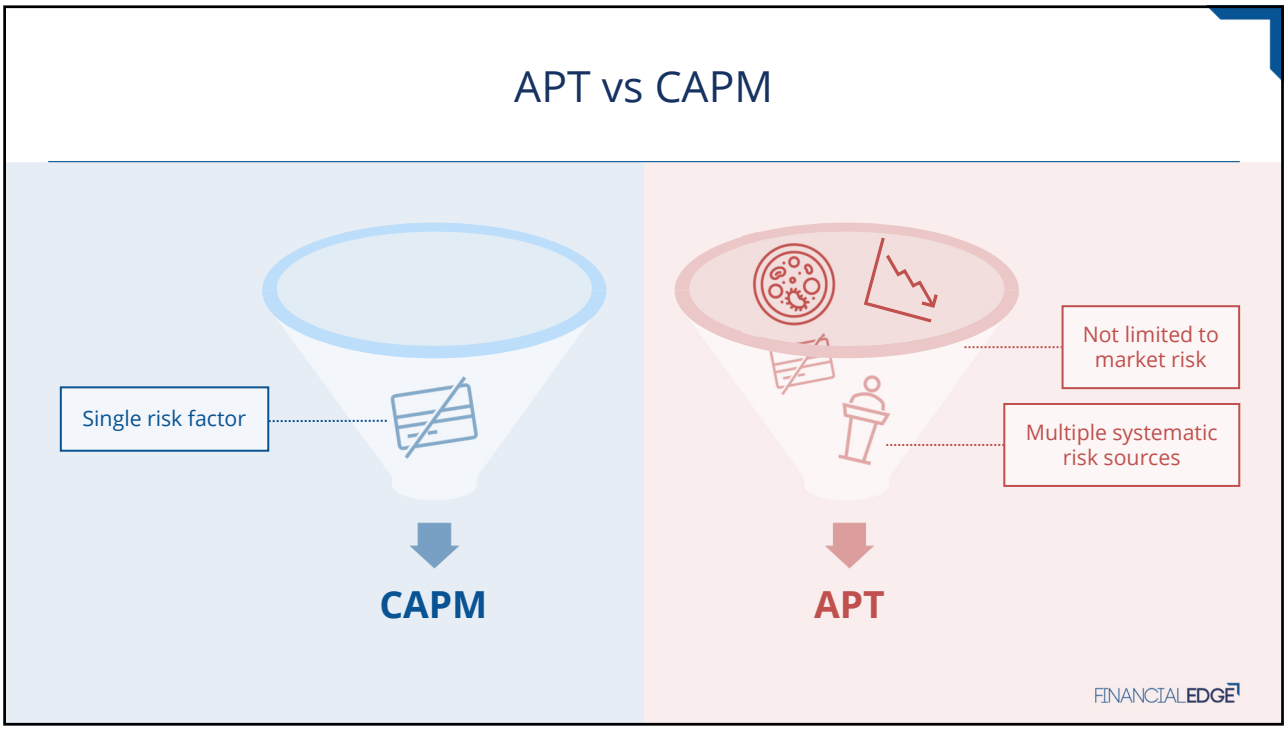
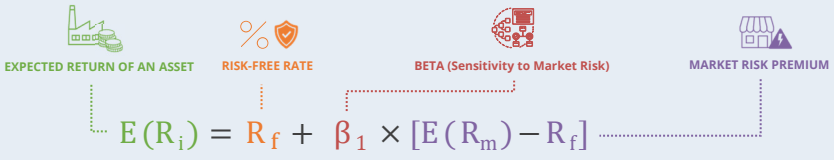
APT vs CAPM

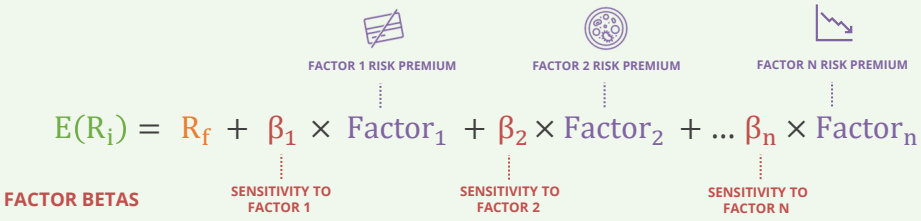


APT vs CAPM

CAPM



APT



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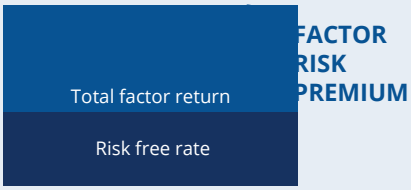
Factor Sensitivity and Factor Returns

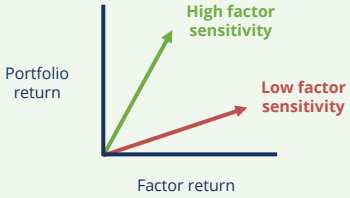
FACTOR RISK PREMIUM



FACTOR SENSITIVITY

Similar to the market beta used in CAPM. It measures the sensitivity of expected return to a specific factor.

Also know as: **Factor Betas**



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APT Example

Assuming a risk-free rate of 2%, calculate the expected return of the security

	FACTOR SENSITIVITY	FACTOR RISK PREMIUM
RISK FACTOR 1	1.27	2.59%
RISK FACTOR 2	0.56	0.66%
RISK FACTOR 3	0.37	4.32%

$$E(R_i) = R_f + \beta_1 \times Factor_1 + \beta_2 \times Factor_2 + \ldots \beta_n \times Factor_n$$

$$E(R_i) = 2\% + (1.27 \times 2.59\%) + (0.56 \times 0.66\%) + (0.37 \times 4.32\%)$$

$$E(R_i) = 7.26\%$$

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APT Risk Factors

Unlike CAPM, **APT does not identify relevant risk factors**

EXPECTED RETURN ON THE ASSET

Risk factors **need to be selected for input into an APT model**

- Market index returns
- Bond yield curve movements
- Default rates
- Inflation
- FX rates
- Commodity prices
- GDP and GNP
- Industrial production
- Others

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Fama-French Three Factor Model

SIMILARITIES TO PRIOR MODELS

- Initial factor is market exposure like CAPM
- Multi-factor model like APT

DIFFERENCES FROM PRIOR MODELS

- Unlike APT, relevant risk factors are specified
- Introduced firm specific micro factors

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Fama-French Three Factor Model

Two groups of stocks tend to have higher returns than those predicted solely by their sensitivity to the market return

SMALL-CAPITALIZATION STOCKS

VALUE STOCKS
(High book-to-market ratio)

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Fama-French Three Factor Model

MARKET RETURN
(EXPOSURE)

SIZE EFFECT

VALUE EFFECT

$$E(R_i) = R_f + (\beta_m \times MRP_M) + (\beta_s \times SMB) + (\beta_v \times HML)$$

Size Risk Premium
Historical differences in returns
(Small Minus Big)

Book-to-Market Premium
Historical differences in returns
(High book/market Minus Low)

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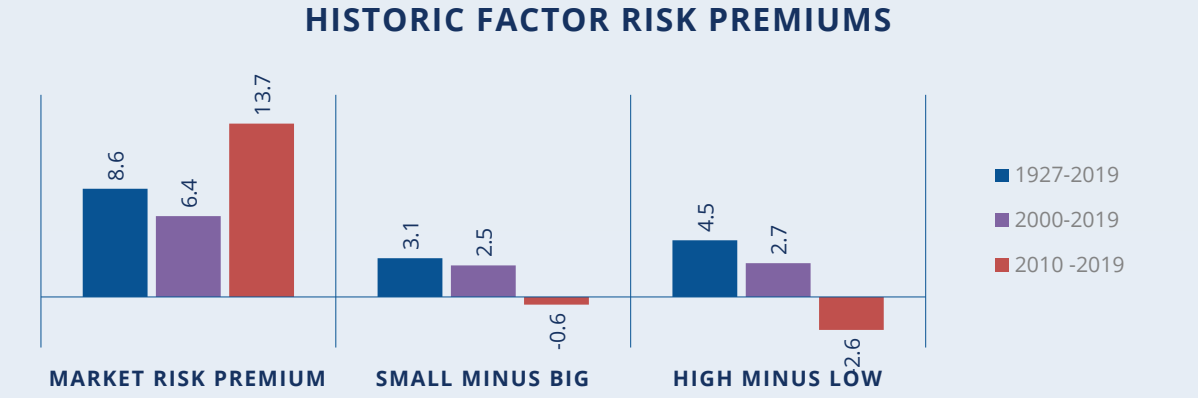
Fama-French Three Factor Model



Source: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html. Average annual values.

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