

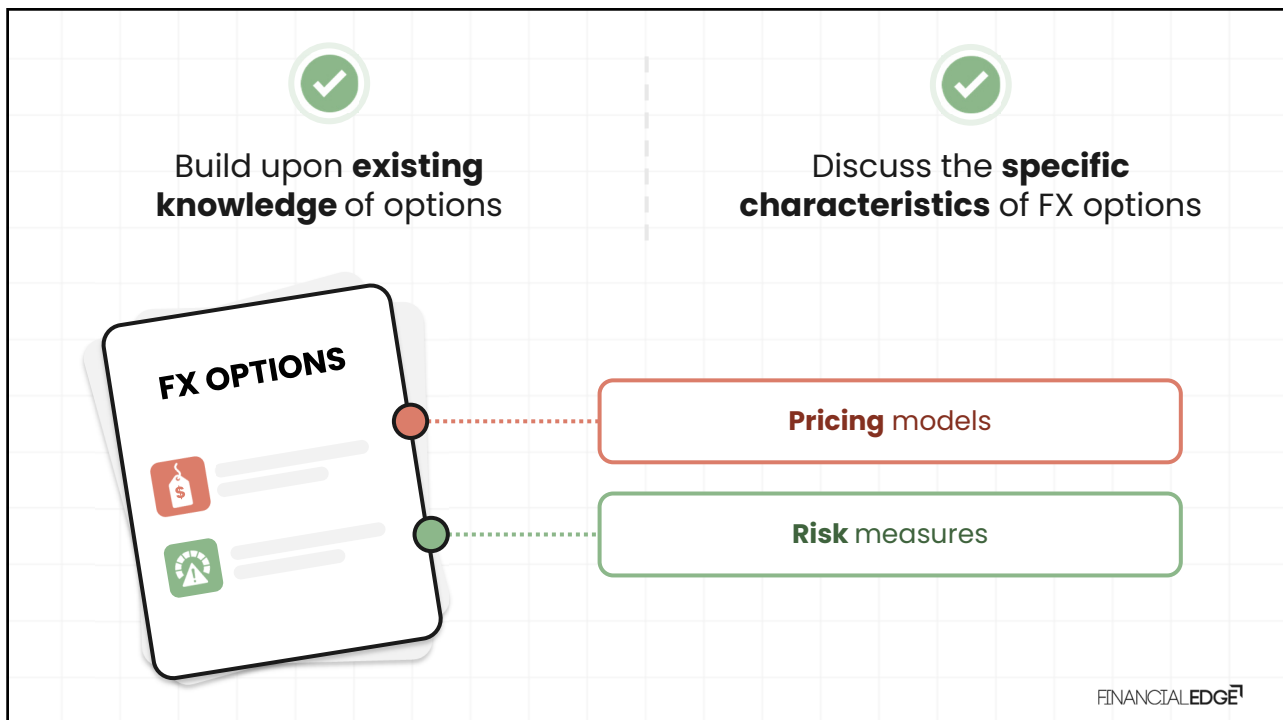
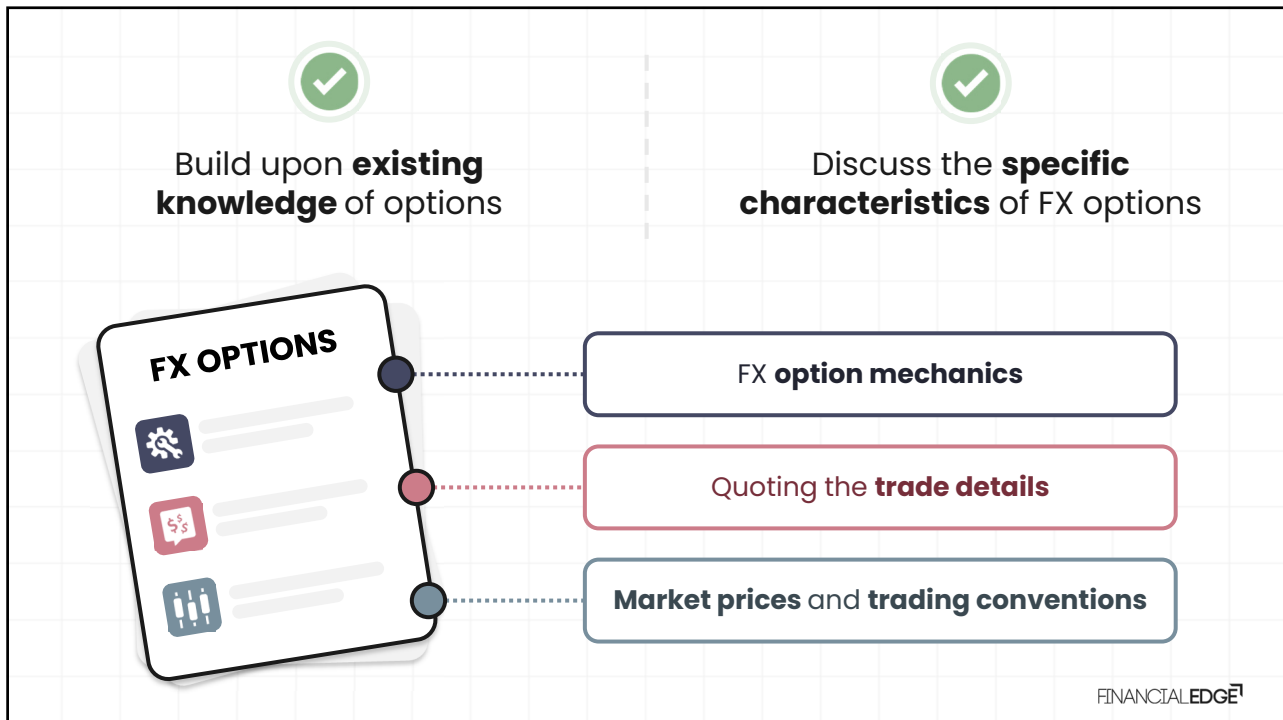


# FX Options



## FX Option Definition, Terminology and Convention





## Definition

The **right to buy** (call) or **sell** (put) an **underlying currency** at a defined **exchange rate** (strike) on a defined **future date** (expiry)



Do I have the right to **buy/sell**?



Which currency is **premium paid**?

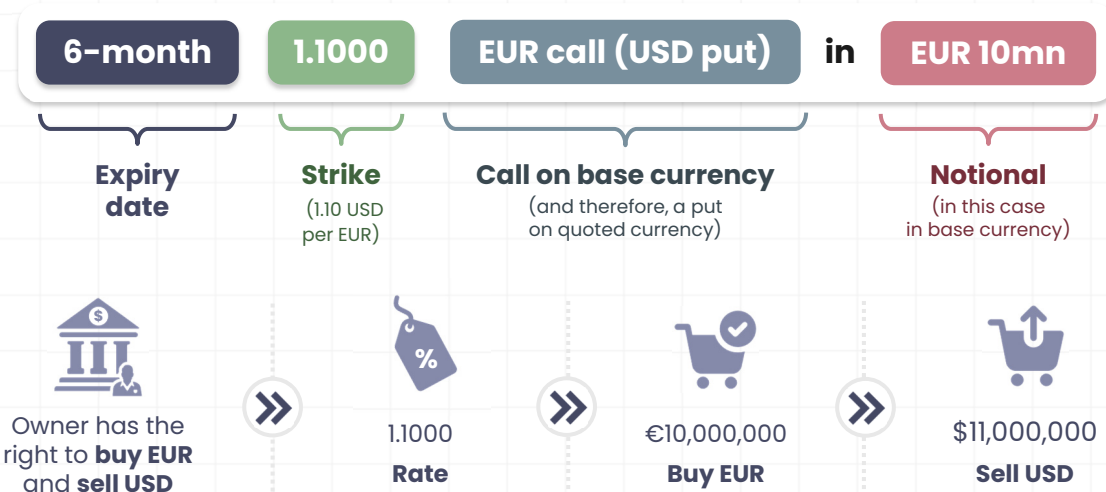
Rather than having an **obvious asset** and **its price**



You have **two cash amounts** in **different currencies**

FINANCIALEDGE<sup>1</sup>

## Example

FINANCIALEDGE<sup>1</sup>

# Terminology

There are **many ways** to refer to the **same FX option**

Referring to the **same FX option**:

EUR call / USD put

EUR call

USD put

EURUSD call

**Quoted** with the **notional** in either currency:

1.1000 EUR call in EUR 10mn

1.1000 USD put in USD 11mn

0.9091 USD put in USD 11mn

(quoted on an **inverted** USDEUR rate)



**NEED market conventions**

FINANCIALEDGE<sup>7</sup>

# Market Conventions



**ALWAYS** describe in terms of the **base currency**

**Notional**

Quoted in **EUR**

**Strike**

Quoted in **conventional** wholesale market format

**EURUSD** (not USDEUR)

**Type**

EUR call/put, or  
EURUSD call/put

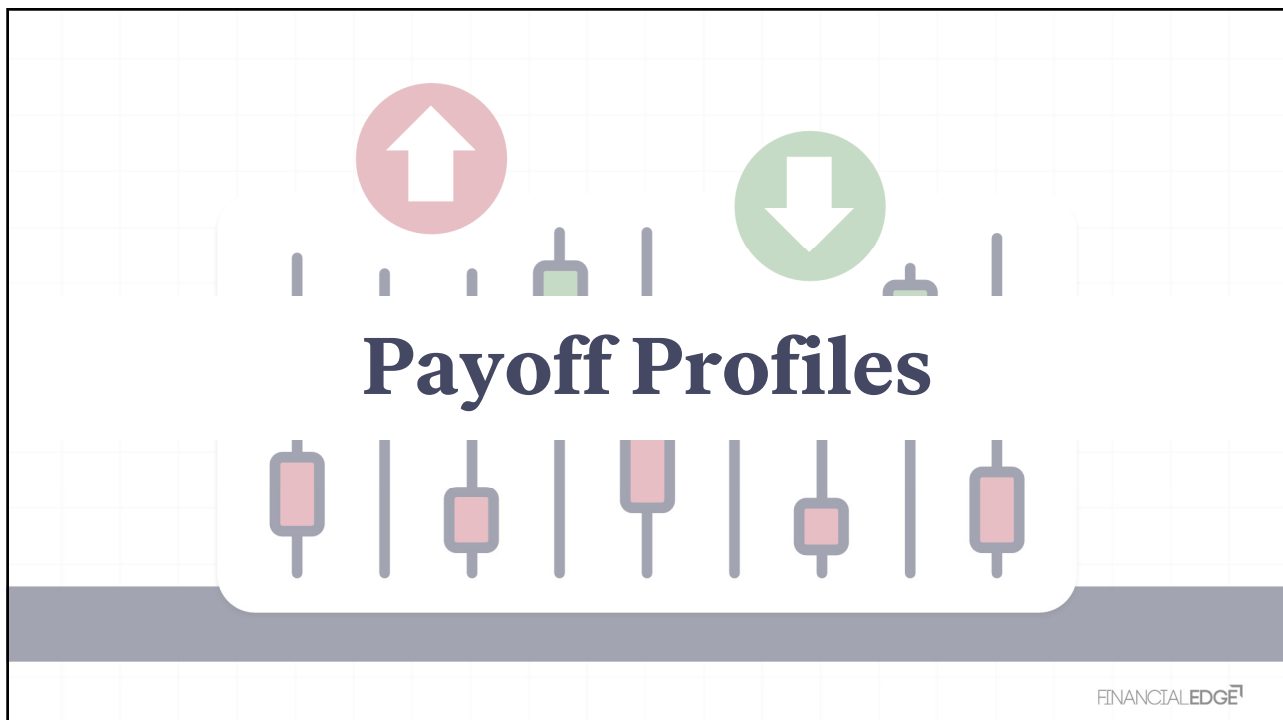
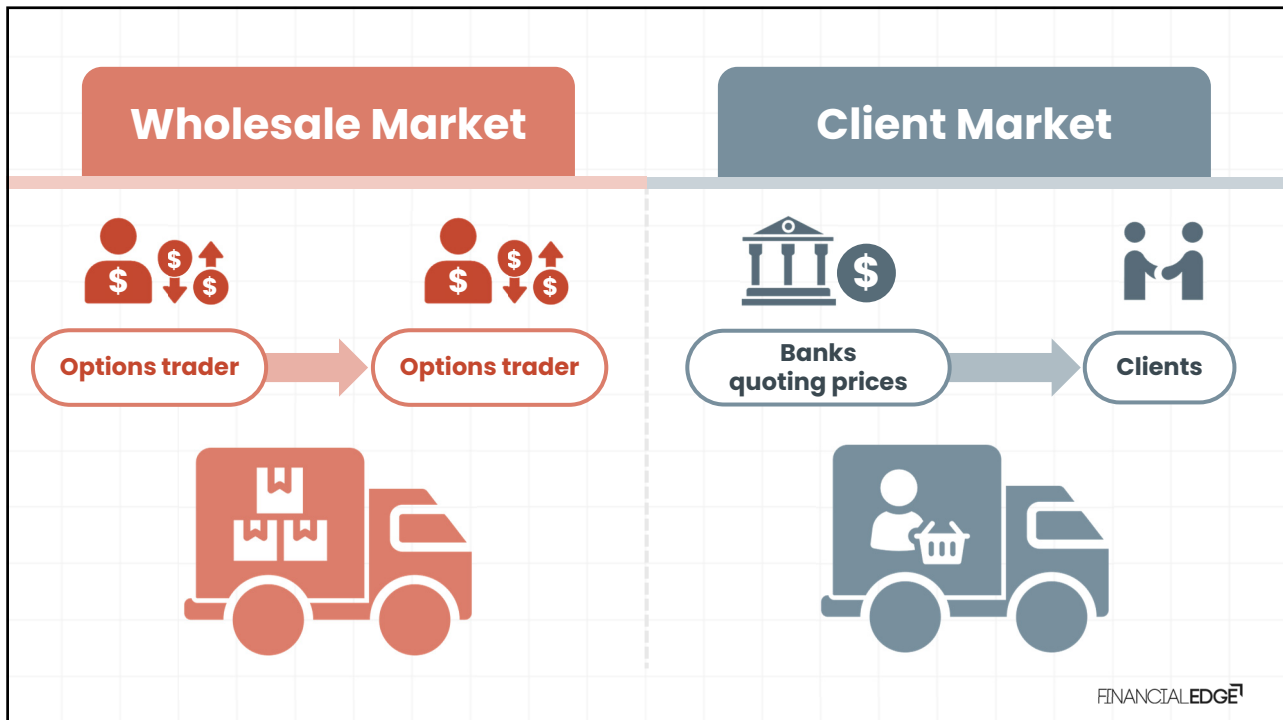
**Non-wholesale market** counterparties may use **any quoting conventions**

"EUR call USD put"

"EUR call"

"EURUSD call"

FINANCIALEDGE<sup>7</sup>





## PAYOFF OF AN OPTION

Calculating what the **positive value** is to the buyer if the option is exercised

### Stock Options



**Own**

100 Call



**Stock**

110



**Payoff**

\$10 per share

FINANCIALEDGE<sup>7</sup>

The payoff of an FX option naturally occurs in the **quoted currency**

#### Call

$$\text{Payoff}_{\text{quoted}} = \text{Max} ( \text{Spot}_{\text{expiry}} - K , 0 ) * N_{\text{base}}$$

#### Put

$$\text{Payoff}_{\text{quoted}} = \text{Max} ( K - \text{Spot}_{\text{expiry}} , 0 ) * N_{\text{base}}$$

FINANCIALEDGE<sup>7</sup>

If we want the payoff in the base currency,  
we **divide** by the spot at **expiry**

Base

Payoff<sub>base</sub>

=

Payoff<sub>base</sub>

Spot<sub>expiry</sub>

FINANCIALEDGE<sup>7</sup>

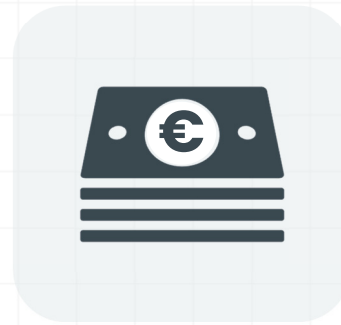
## Payoff in Quoted Currency and Base Currency

FINANCIALEDGE<sup>7</sup>

## Payoff Examples



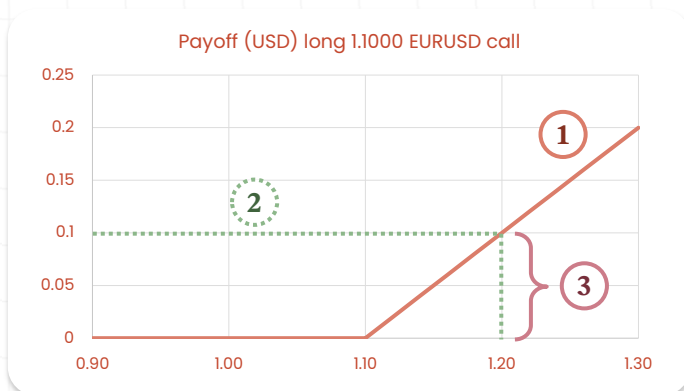
**Quoted Currency**



**Base Currency**

FINANCIALEDGE<sup>7</sup>

## Quoted Currency

**1**

**Payoff is linear in  
quoted currency**

**2**

**Example:  
Spot finishes at 1.2000**

**3**

**Payoff = +0.1 USD per EUR**  
**BUY spot @ 1.1000**  
**SELL @ 1.2000**

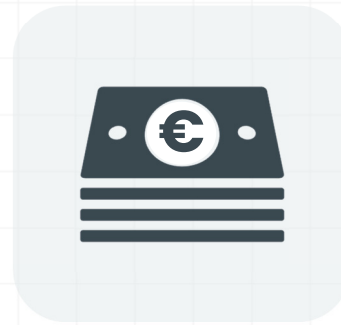
FINANCIALEDGE<sup>7</sup>



## Payoff Examples



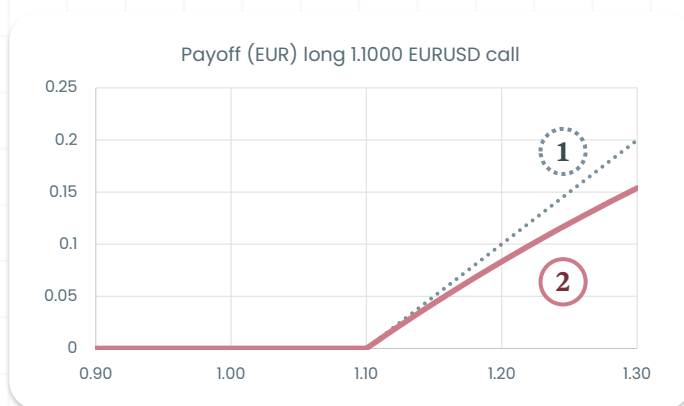
**Quoted Currency**



**Base Currency**

FINANCIALEDGE<sup>7</sup>

## Base Currency



1

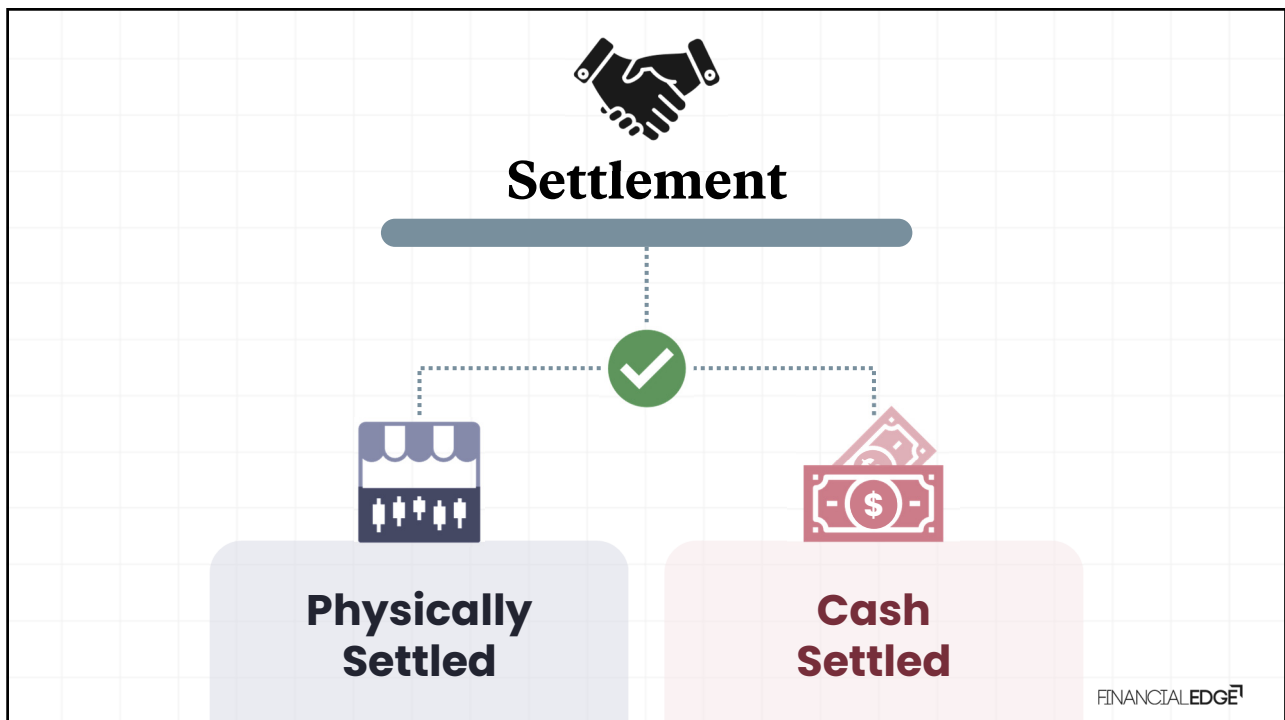
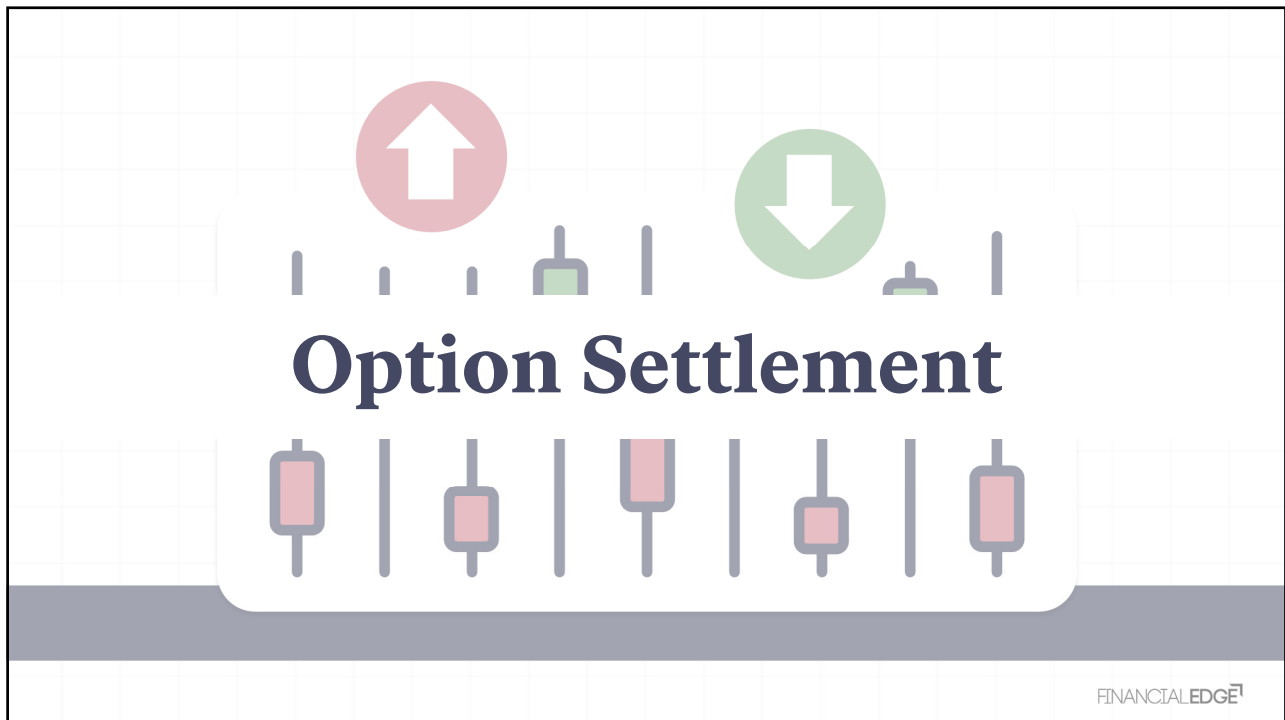
Payoff is **non-linear**  
in **base** currency

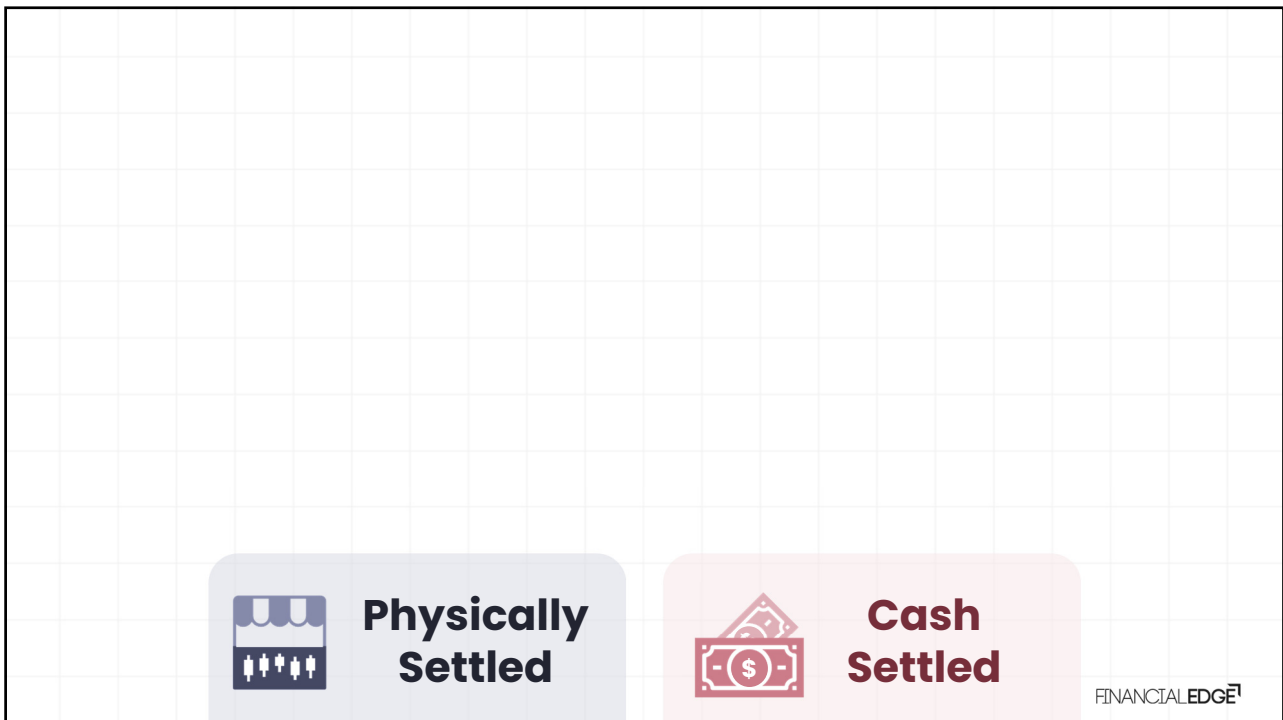
2

The equivalent in  
**stock options** would be  
**calculating the payoff in**  
**shares** rather than cash

Simpler to think of linear  
payoff in **quoted** currency

FINANCIALEDGE<sup>7</sup>





The payoff is calculated relative to an agreed fixing source and paid in either **quoted** or **base** currency



**Own**

138.00 USDJPY



**Put in**

USD 10,000,000



**Spot fixing on expiry**

135.00



**Physically  
Settled**



**Cash  
Settled**

FINANCIALEDGE<sup>7</sup>

The payoff is calculated relative to an agreed fixing source and paid in either **quoted** or **base** currency

$$\text{Payoff(JPY)} = 10,000,000 * (138 - 135) = \text{JPY } 30,000,000$$

$$\text{Payoff(USD)} = 10,000,000 * (138 - 135) / 135 = \text{USD } 222,222.22$$

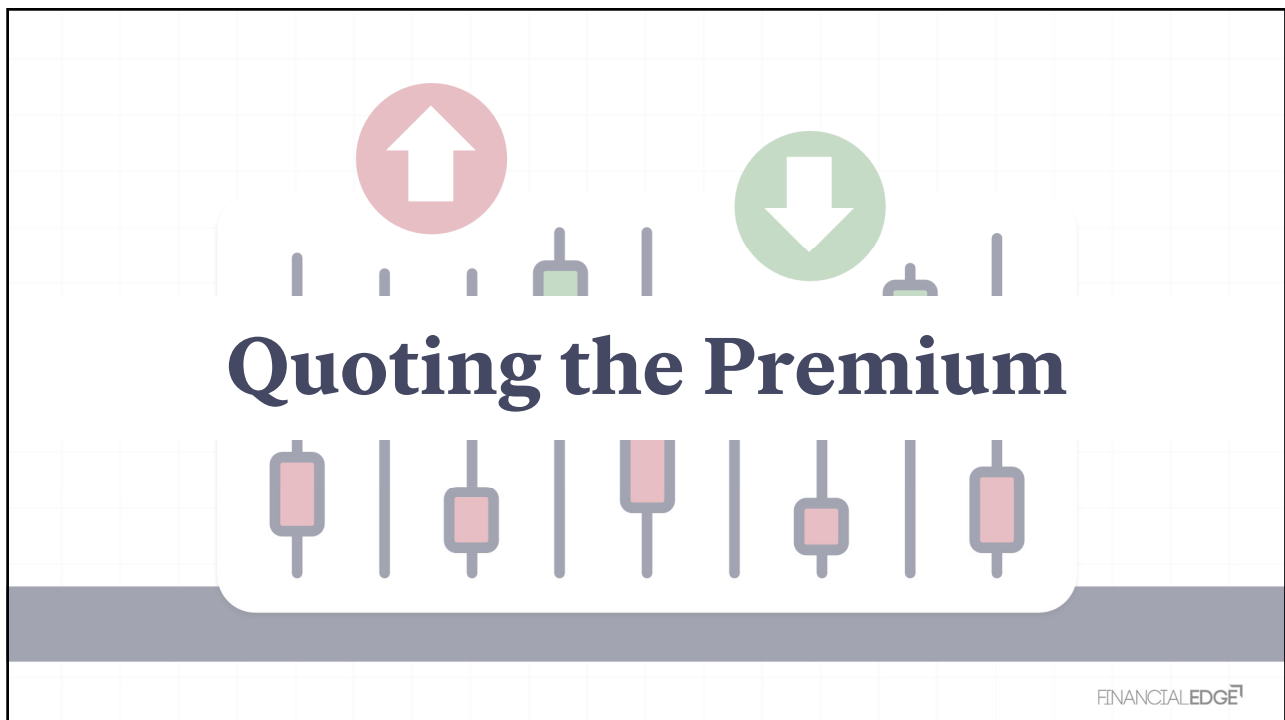
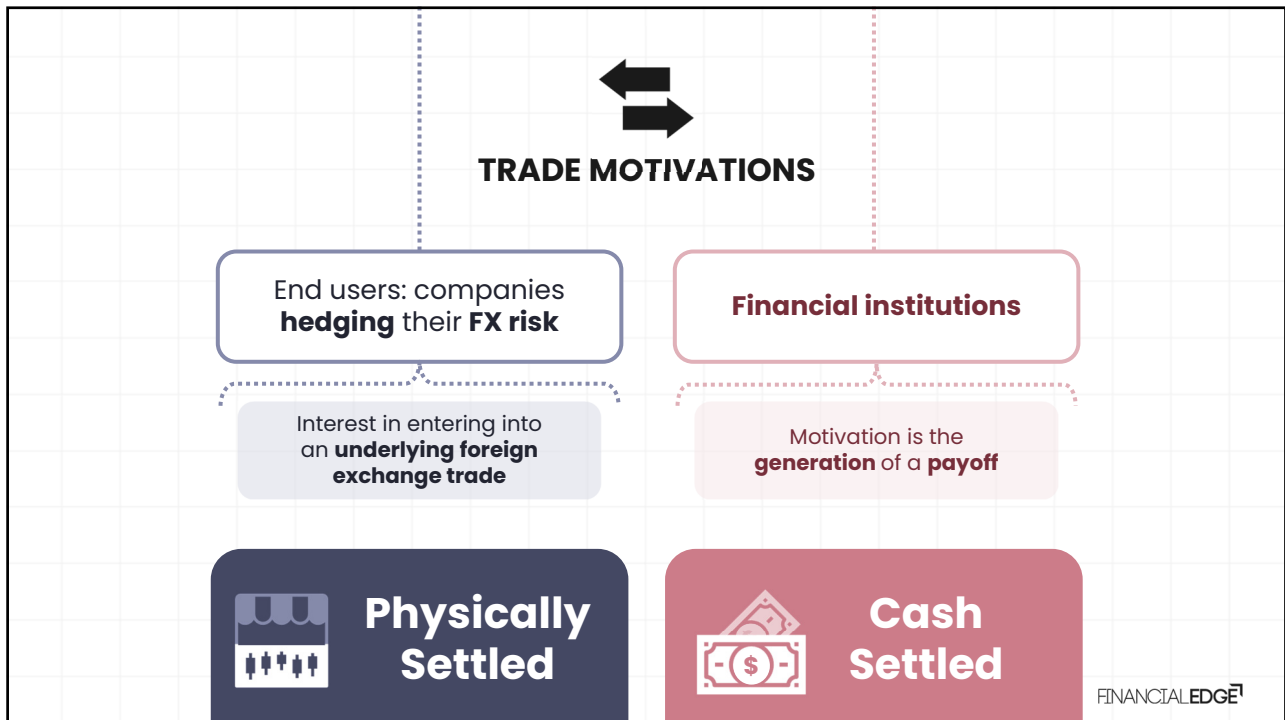


**Physically  
Settled**



**Cash  
Settled**

FINANCIALEDGE<sup>7</sup>





## The Premium

Option pricing models (e.g. Black-Scholes) naturally produce a price for an FX option in the **quoted currency**

### 3-month EURUSD 1.1000 call

R(base)	R(quoted)	Expiry	Spot	Forward	Strike	Type	Vol	Price (quoted)
3.50%	5.00%	0.25	1.0850	1.1041	1.1000	c	10.00%	0.0166

Price = 0.0166 USD per EUR of notional, or **166 USD pips**

### 3-month EURUSD 1.1000 call

R(base)	R(quoted)	Expiry	Spot	Forward	Strike	Type	Vol	Price (quoted)
3.50%	5.00%	0.25	1.0850	1.1041	1.1000	c	10.00%	0.0166

Price = 0.0166 USD per EUR of notional, or **166 USD pips**

## Breakeven

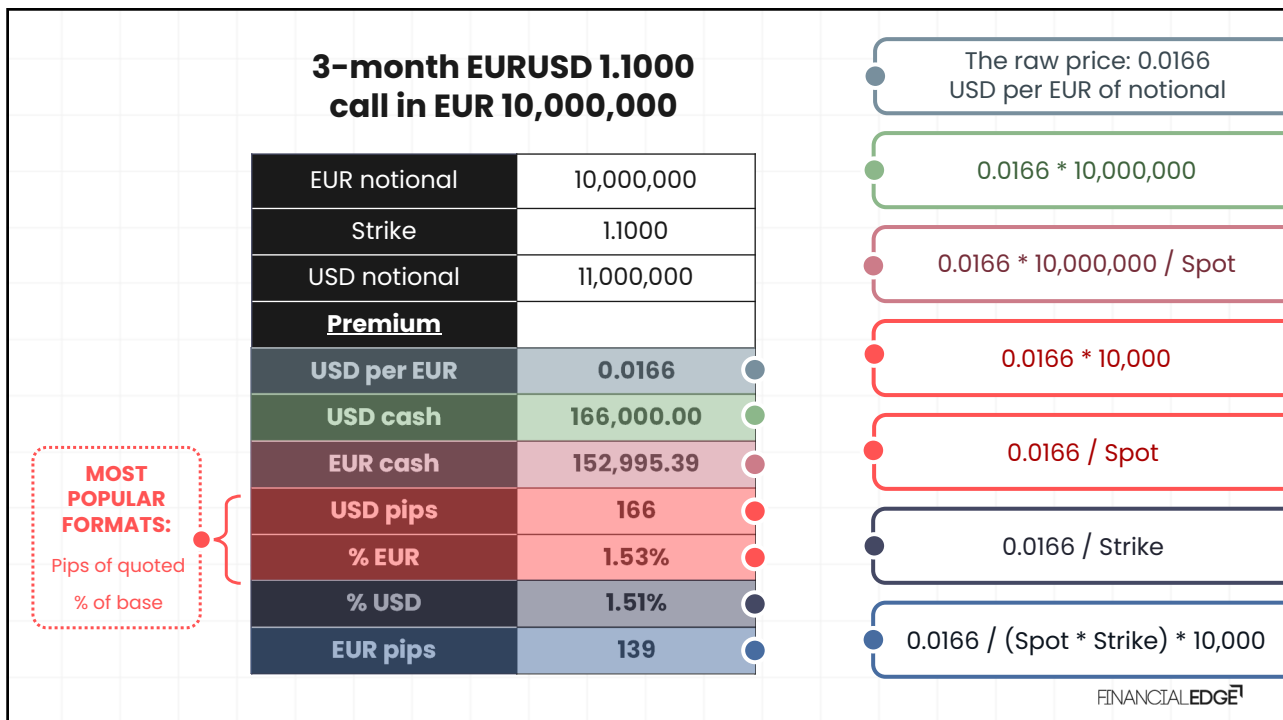
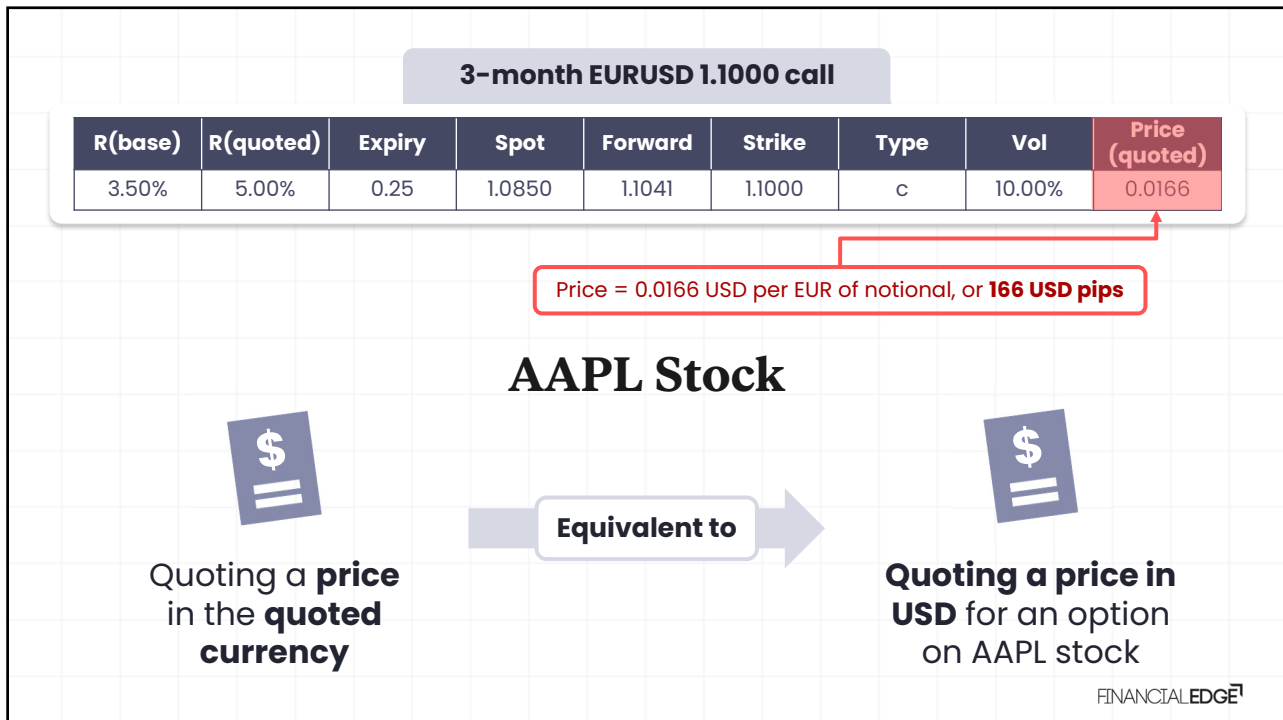


**Buyer**

+/-



**Added onto  
its strike**





## Quoting the Wholesale Market

FINANCIALEDGE<sup>7</sup>

### Wholesale Market

The **interbank market** and bank to **financial institution**



Hedge fund



Quoted in volatility



Black-Scholes pricer

FINANCIALEDGE<sup>7</sup>





## Wholesale Market

### FX Forward Market

- ✓ Very liquid
- ✓ Tight bid offer spreads
- ✓ No uncertainty about price

### Volatility prices not common in other markets

- ✗ Equities
- ✗ Rates

FINANCIALEDGE<sup>7</sup>

## Wholesale Market

Example of price **run of at-the-money** straddles:

1w		1m		2m		3m		6m		9m		12m	
Bid	Offer	Bid	Offer	Bid	Offer	Bid	Offer	Bid	Offer	Bid	Offer	Bid	Offer
8.50	9.50	9.30	9.40	9.35	9.45	9.20	9.30	9.15	9.20	8.90	9.10	8.95	9.00

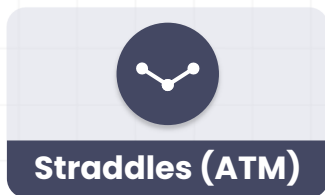
e.g. the market is offering the **3-month expiry** at **9.30% vol**

FINANCIALEDGE<sup>7</sup>



## Wholesale Market

Prices **quoted** as:

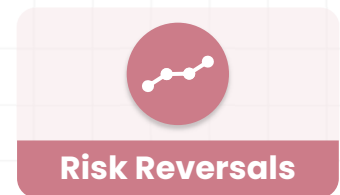


**Straddles (ATM)**



**Butterflies**

25 and 10 delta most common



**Risk Reversals**

25 and 10 delta most common

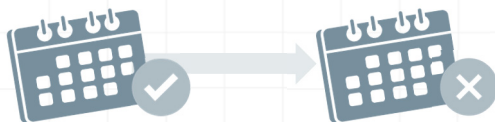
Note use of a **delta moneyness** reference instead of **strike** – common in **FX options**

FINANCIALEDGE<sup>7</sup>



## Wholesale Market

**0-1 YEAR**



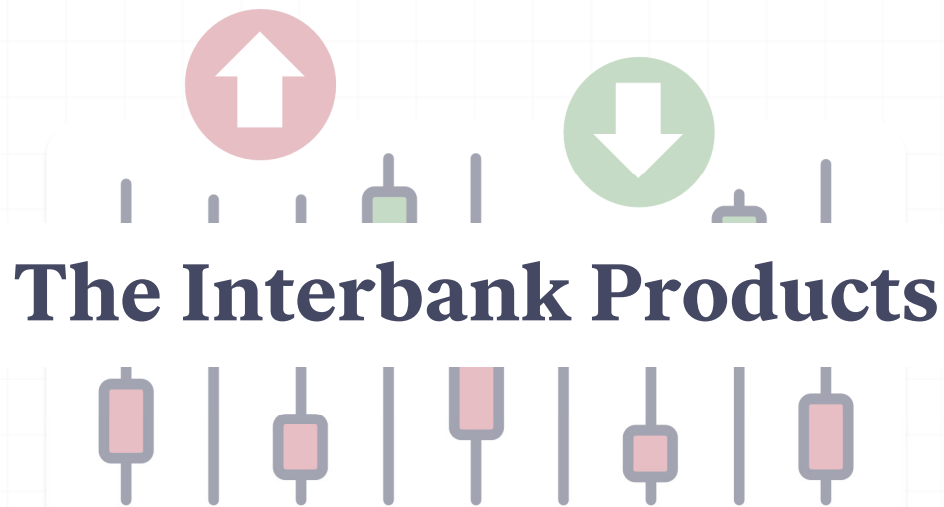
**Good liquidity**

**>1 YEAR**



**Liquidity diminishes**

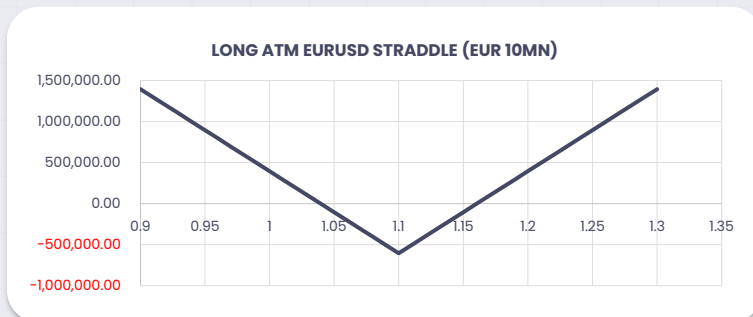
FINANCIALEDGE<sup>7</sup>





## Straddles

Most common **interbank** trading product



### Combination of:

ATM put and ATM call

ATM = AT THE MONEY

Long both or short both

### Allows traders to:

Trade volatility **without** getting a **delta position**

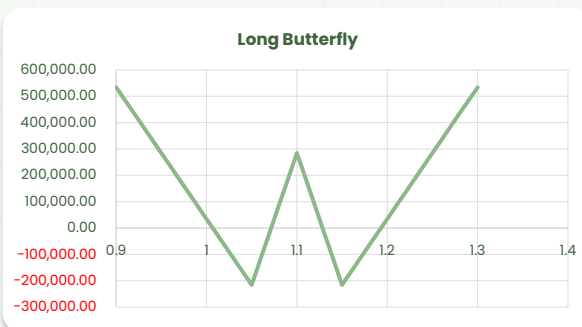
FINANCIALEDGE<sup>7</sup>



## Butterflies

**Long strangle + short straddle** (or vice versa)

Tend to be done in **vega-neutral** amounts



## Risk Reversal

**Long call + short put** (or vice versa)



FINANCIALEDGE<sup>7</sup>

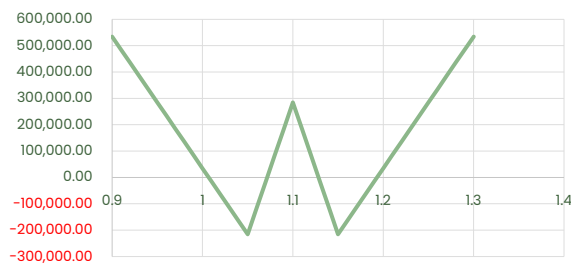


## Butterflies

Trade the **smile element**

Quoted in **delta terms**

Long Butterfly

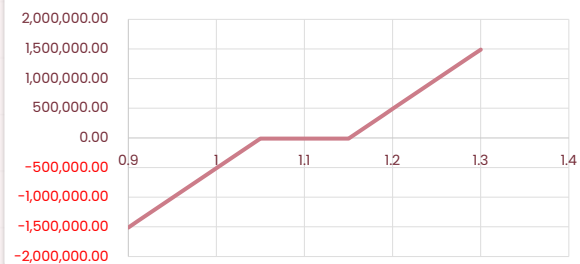


## Risk Reversal

Trade the **skew**

Quotes in **delta terms**

Risk Reversal



FINANCIALEDGE<sup>7</sup>



## Quoting Butterflies and Risk Reversals in Vol



FINANCIALEDGE<sup>7</sup>



## Butterflies



## Risk Reversal

FINANCIALEDGE<sup>7</sup>

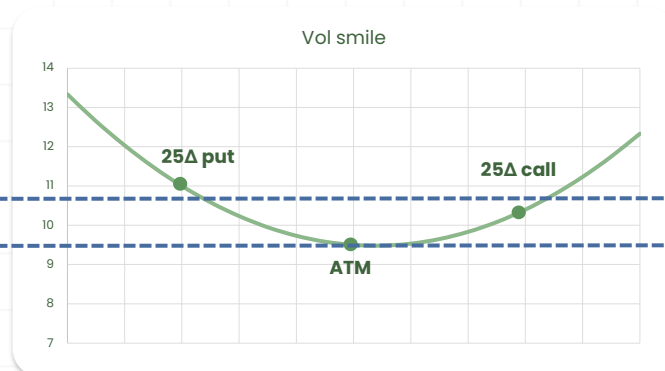
## Butterflies

$$25\Delta \text{ Butterfly vol} = \frac{25\Delta \text{ call vol} + 25\Delta \text{ put vol}}{2} - \text{ATM vol}$$

25Δ average = 10.75

ATM = 9.50

ATM = AT THE MONEY

FINANCIALEDGE<sup>7</sup>



## Butterflies

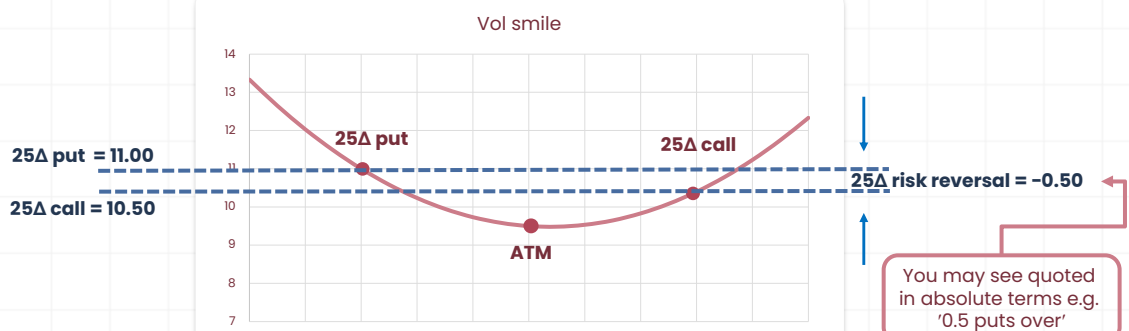


## Risk Reversal

FINANCIALEDGE<sup>7</sup>

## Risk Reversal

$$25\Delta \text{ Risk Reversal vol} = 25\Delta \text{ call vol} - 25\Delta \text{ put vol}$$



ATM = AT THE MONEY

FINANCIALEDGE<sup>7</sup>

## Volatility for Strike

Use the **interbank quotes** to get the vols for the **commonly quoted points**, that is **25- and 10-delta strikes**

Vol	
ATM	8.00
25d BF	0.75
25d RR	-0.50

**Call**

$$25\Delta \text{ call vol} = \text{ATM} + 25\Delta \text{ BF} + \left( \frac{25\Delta \text{ RR}}{2} \right)$$

$$25\Delta \text{ call vol} = 8.00 + 0.75 - 0.25 = 8.50$$

**Put**

$$25\Delta \text{ put vol} = \text{ATM} + 25\Delta \text{ BF} - \left( \frac{25\Delta \text{ RR}}{2} \right)$$

$$25\Delta \text{ put vol} = 8.00 + 0.75 + 0.25 = 9.00$$

ATM = AT THE MONEY

BF = BUTTERFLIES

RR = RISK REVERSAL

FINANCIALEDGE<sup>7</sup>

## Modelling Choices



FINANCIALEDGE<sup>7</sup>



## Standard Lognormal Approach

FX options tend to use the standard **lognormal** distributional assumption in the **Black-Scholes** formula

### Call Price

$$\text{Call Price} = \text{PV} \left( F * N(d1) - K * N(d2) \right)$$

$$d1 = \frac{\text{LN} \left( \frac{F}{K} \right) + \sigma^2 T}{\sigma \sqrt{T}}$$

$$d2 = d1 - \sigma \sqrt{T}$$

FINANCIALEDGE<sup>7</sup>

As per other markets, **corrections** are then made by using a **volatility smile/skew**



### Smile

All vols for **out-of-the-money options** tend to be **higher** than the **at-the-moneys**

To adjust for **stochastic volatility**



### Skew

Vols will display a **systematic bias** to puts or calls

To adjust for **spot/vol correlation**

FINANCIALEDGE<sup>7</sup>



## Calibrating the Vol Smile

FINANCIALEDGE<sup>7</sup>

## Calibration of the Volatility Smile and Skew

### SABR Calibration

BETA

ALPHA

RHO

FINANCIALEDGE<sup>7</sup>

## Calibration of the Volatility Smile and Skew

### BETA

$$dF = \sigma F^{\beta} dW_1$$

$\beta$  = CEV parameter (skew)

CEV = Constant Elasticity of Variance

### ALPHA

$$d\sigma = \alpha \sigma dW_2$$

$\alpha$  = vol of vol (smile)

Can be varied between **zero** and **one**

**Beta = 0** – Normal distribution  
**Beta = 1** – Lognormal distribution

### RHO

$$dW_1 dW_2 = \rho dt$$

$\rho$  = correlation between the two processes (skew)

Used to further **refine the skew**

Calibrated to **match the skew** of visible risk reversal prices

FINANCIALEDGE<sup>7</sup>

### FLAT

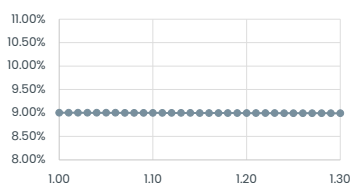
$$dF = \sigma F^{\beta} dW_1$$

**Black-Scholes** assumption

**Lognormal** process

**Constant** vol per strike

$\alpha = 0, \beta = 1, \rho = 0$



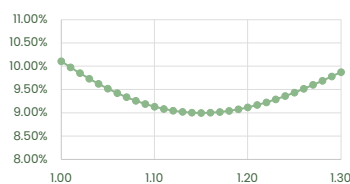
### SMILE

$$d\sigma = \alpha \sigma dW_2$$

'Smiley' volatility shape created by increasing **Alpha**

**Non-zero butterfly prices** but **neutral risk reversals**

$\alpha = 0.6, \beta = 1, \rho = 0$



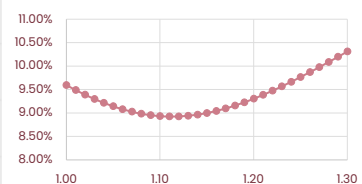
### SMILE AND SKEW

$$dW_1 dW_2 = \rho dt$$

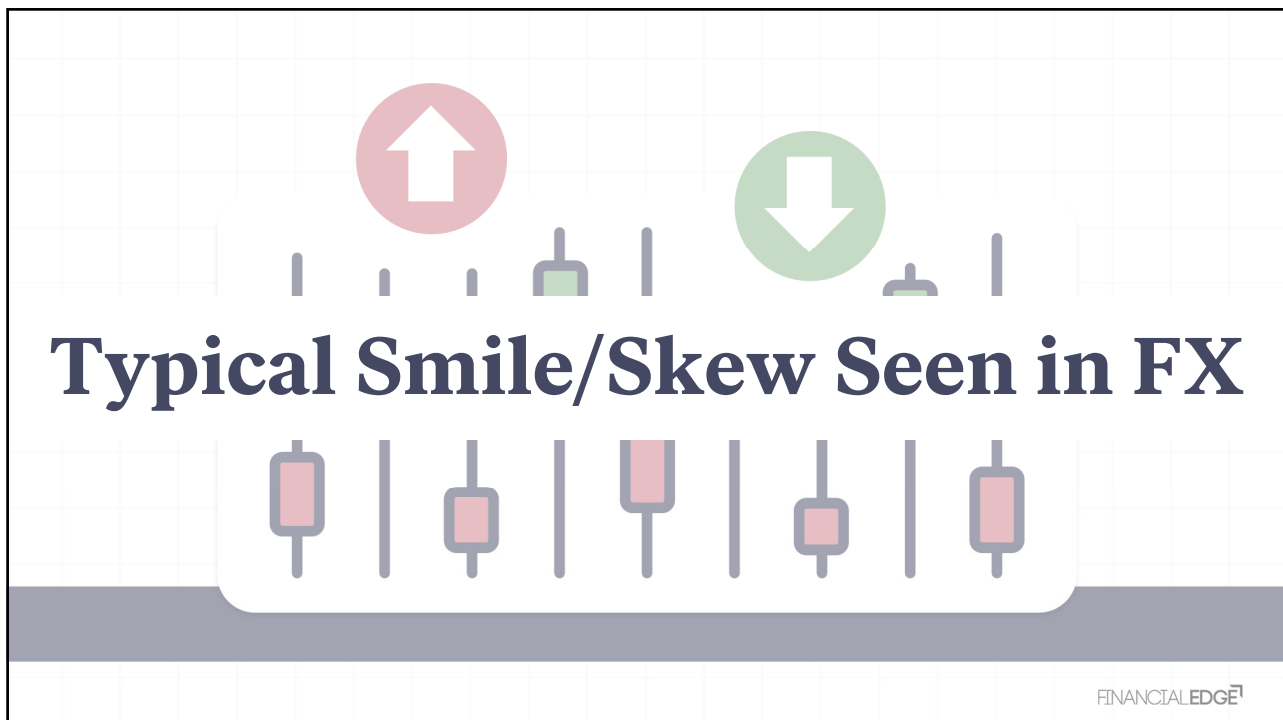
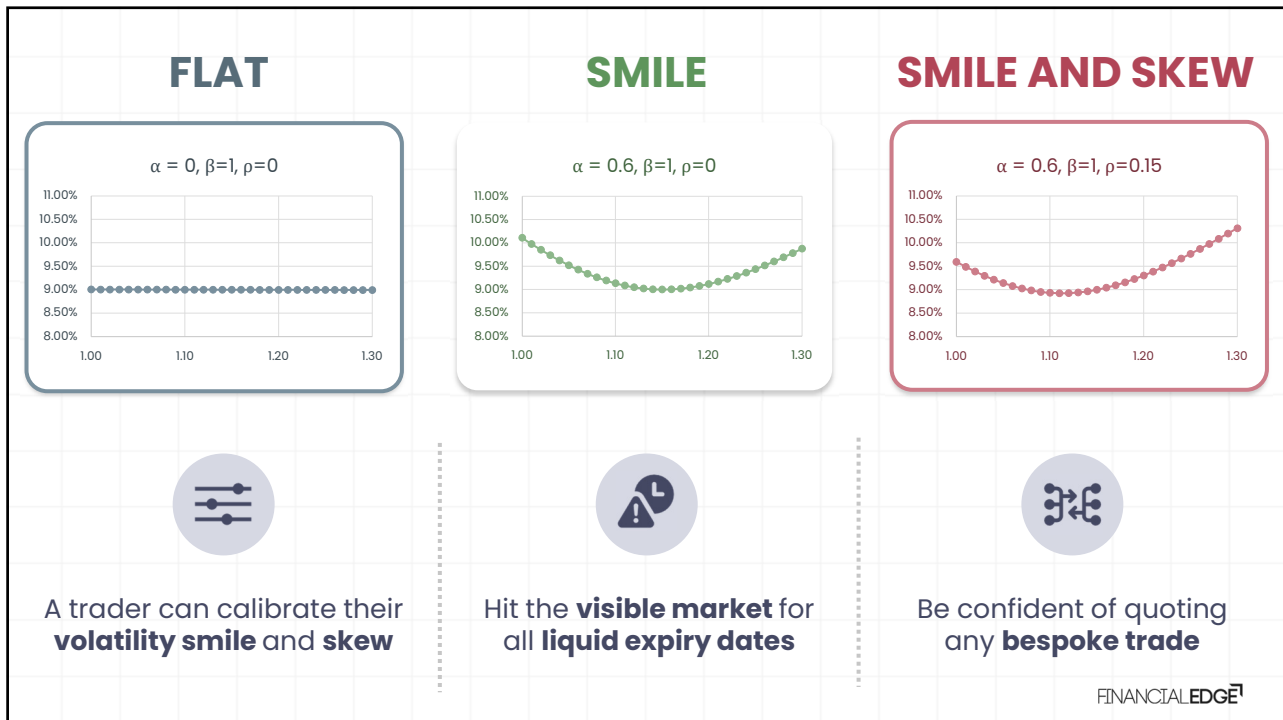
Effect on **increasing Rho**

Adds an **increase to call over puts** whilst retaining the **smiley-ness** already present

$\alpha = 0.6, \beta = 1, \rho = 0.15$



FINANCIALEDGE<sup>7</sup>





## What we **DON'T** see



### Rates

The tendency to use a **normal** rather lognormal **distribution**

**Strong and persistent bid for puts over calls** in risk reversals



### Equities

Commonly **negative correlation** between **spot** and **implied vol**

**Market moves** down are accompanied by a **rise in volatility**

**Supply and demand effects** from end user trading that can keep the **skew bias** in place

FINANCIALEDGE<sup>7</sup>



## What we **DO** see

FX options generally **more balanced** in terms of **skew**



Risk reversals **favor puts** in **certain circumstances**



Also common to **favor calls** depending on **market conditions**



More 'smiley rather than **skewed**' compared to both **equities** and **rates**

FINANCIALEDGE<sup>7</sup>

## Asymmetric Event Risk

Persistent skew can develop if there is an event that contributes to a **significant correlation** between movements in **spot** and **implied vol**



Move spot **gently back** one way



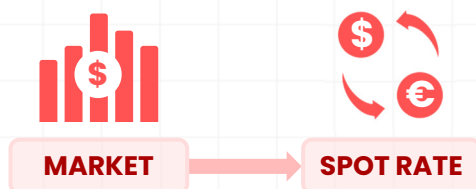
**Calming effect** on the market



FINANCIALEDGE<sup>7</sup>



### Swiss National Bank

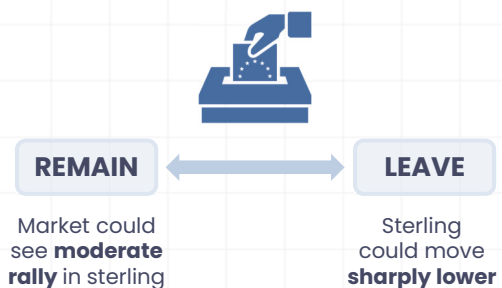


**Dramatic move lower**

**EURCHF**



### EU Referendum



**GBPUSD**

FINANCIALEDGE<sup>7</sup>



The graphic features a central title 'FX Option Risk Metrics' in a dark blue serif font. Above the title are two large circular icons: a red one with a white upward arrow and a green one with a white downward arrow. Below the title are several vertical lines and small rectangular blocks in red and green, resembling a stylized bar chart or candlestick pattern. The entire graphic is set against a light gray grid background.

## FX Option Risk Metrics

FINANCIALEDGE<sup>7</sup>

### Risks Generated on a 6-month Call and Put

			<u>1.10 call</u>	<u>1.10 put</u>
<b>R(quoted)</b>	5.00%	<b>Price</b>	0.0286	0.0205
<b>R(base)</b>	3.50%	<b>Delta (spot)</b>	0.55412	-0.42853
<b>Expiry</b>	0.50	<b>Delta (fwd)</b>	0.56390	-0.43610
<b>Spot</b>	1.1000	<b>Gamma (spot)</b>	6.19394	6.19394
<b>Forward</b>	1.1083	<b>Gamma (fwd)</b>	6.25657	6.25657
<b>Strike</b>	1.1000	<b>Vega</b>	0.00301	0.00301
<b>Vol</b>	8.00%	<b>Theta</b>	-0.00006	-0.00006

**Strike**

1.10

**Spot**

1.10

**Forward**

1.1083

**Vol**

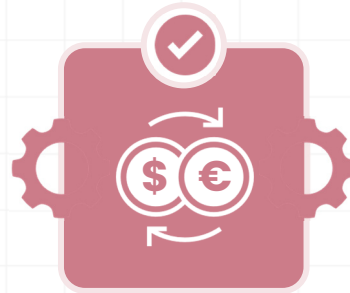
8.0%

FINANCIALEDGE<sup>7</sup>

## Convert into...



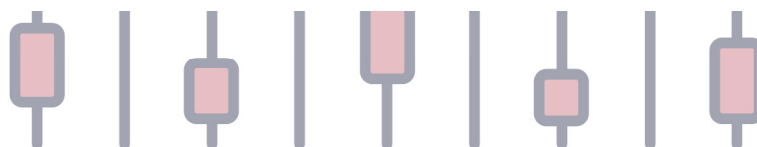
Cash figures



Equivalent spot  
FX positions

FINANCIALEDGE<sup>7</sup>

## Delta

FINANCIALEDGE<sup>7</sup>



## Converting FX Option Risk Metrics



Delta



Gamma



Vega and Theta



### Delta

It is **conventional** for an FX Options trader to **hedge their delta** using **spot**

It may be deemed **purer to measure delta to the forward rate**



Spot FX is **easier to trade and hedge**



The exposure to **forward points** may be **hedged less frequently**

Extra Greeks, usually  $\rho$  and  $\epsilon$ , are used to measure sensitivity to  $R_q$  and  $R_b$  respectively



## Delta

To convert the raw Black-Scholes delta into **something more useful**



### SPOT FX AMOUNT

	1.10 call
Delta (spot)	0.55412
Notional (EUR)	10,000,000
Delta (EUR)	5,541,184

If you **BUY** the **1.10 call** in **EUR 10mn**

You are long **EUR 5.5mn** of **spot delta**

FINANCIALEDGE<sup>7</sup>

## Delta

Incorporate the **premium** into **(initial)** delta hedge

If you don't **convert premium back** into the **currency** in which you account for your **P/L**, then you are **running a spot FX risk**

	1.10 Put
Notional (EUR)	10,000,000
Price	0.0205
Price (USD Pips)	205
Price (USD)	204,742
Price (%EUR)	1.86%
Price (EUR)	186,129
Delta (Spot)	-0.42853

### Client...

1



**BUYS** the option

2



**PAYS** the premium in **EUR**

3



Your P/L currency is **USD**

FINANCIALEDGE<sup>7</sup>



## Delta



Need to account for the fact you have an **extra amount of EUR to sell**:

	<u>1.10 put</u>
<b>Notional (EUR)</b>	-10,000,000
<b>Raw Delta (EUR)</b>	4,285,338
<b>Prem (EUR)</b>	186,129
<b>Full Delta (EUR)</b>	4,471,468

To **hedge the option** and **convert the premium** into **USD**

Need to **sell EUR 4.5mn**

FINANCIALEDGE<sup>7</sup>

## What is the ATM Strike?

FINANCIALEDGE<sup>7</sup>

## ATM Strike

There are three ways deciding the at-the-money strike in FX options

### ATMS

At-the-money-spot

**Not common** in FX

### ATMF

At-the-money-forward

Commonly used in trades with **clients**

Call and put have **the same price**, but **different deltas**

### ATM (DNS)

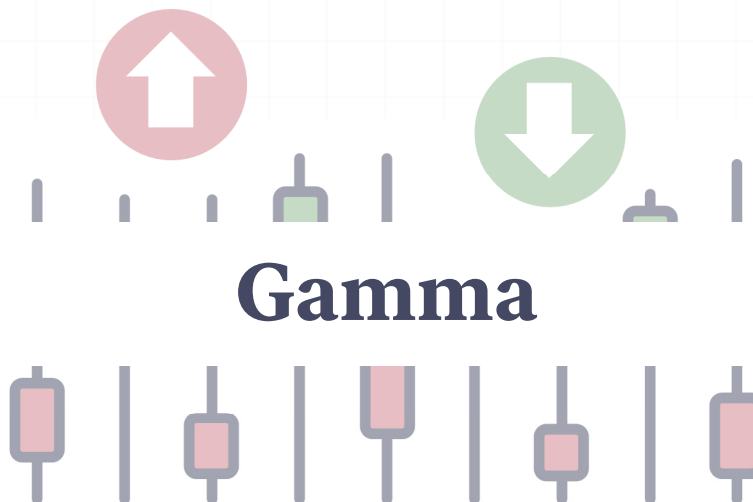
Delta-neutral straddle strike

Call and put have the **same delta**

Standard for **interbank** trading

FINANCIALEDGE<sup>7</sup>

## Gamma



FINANCIALEDGE<sup>7</sup>

## Converting FX Option Risk Metrics



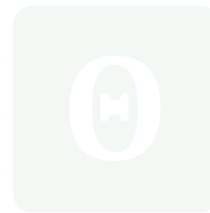
Delta



Gamma



Vega and Theta



# γ

## Gamma

Gamma tells us how **delta changes** and the **delta of an option** can only be between **0 and 1**

	<u>1.10 call</u>	<u>1.10 put</u>
<b>Delta (spot)</b>	0.55412	-0.42853
<b>Gamma (spot)</b>	6.19394	6.19394

**Delta** can change by **6.2**

**Abs (delta)** is bound  $0 < \Delta < 1$

Gamma is trying to 'predict' a delta change for a **1.0 change in spot** using the **local delta gradient**

Scale gamma to a **realistic spot move** and **turn** it into an **equivalent spot FX position change**

$\gamma$ 

## Gamma

FX options traders are likely to want to see their gamma as a change in their **spot FX delta position**

	<u>1.10 call</u>
Notional (EUR)	10,000,000
Gamma (per big figure)	0.06194
Delta change (per big figure)	619,394

Black-Scholes gamma multiplied by 0.01

The **trader** now knows that as **spot moves**

±0.01

**Delta position** changes by

EUR ±0.6mn

FINANCIALEDGE<sup>7</sup> $\gamma$ 

## Gamma

FX options traders are likely to want to see their gamma as a change in their **spot FX delta position**

	<u>1.10 call</u>
Notional (EUR)	10,000,000
Gamma (per big figure)	0.06194
Delta change (per big figure)	619,394

Black-Scholes gamma multiplied by 0.01



**Change the increment**  
(maybe to 10 pips)

OR



Run a **table of delta versus spot**

FINANCIALEDGE<sup>7</sup>

Below is a **delta versus spot** table for a **long 6-month 1.1000 EURUSD straddle in EUR 10,000,000 (@8% vol)**

Spot	P/L	Delta	Gamma (per big figure)
1.1500	139,783	5,186,019	769,483
1.1450	114,840	4,786,467	828,711
1.1400	91,968	4,357,383	887,486
1.1350	71,314	3,899,187	945,021
1.1300	53,023	3,412,707	1,000,482
1.1250	37,232	2,899,195	1,053,007
1.1200	24,074	2,360,335	1,101,724
1.1150	13,668	1,798,249	1,145,773
1.1100	6,126	1,215,478	1,184,331
1.1050	1,543	614,961	1,216,634
1.1000	0	0	1,242,003
1.0950	1,561	-625,789	1,259,862
1.0900	6,269	-1,258,535	1,269,765
1.0850	14,151	-1,894,177	1,271,408
1.0800	25,209	-2,528,541	1,264,644
1.0750	39,427	-3,157,422	1,249,490
1.0700	56,767	-3,776,664	1,226,135
1.0650	77,171	-4,382,248	1,194,932
1.0600	100,561	-4,970,372	1,156,392
1.0550	126,840	-5,537,526	1,111,171
1.0500	155,896	-6,080,560	1,060,052

FINANCIALEDGE<sup>7</sup>

## Vega and Theta

FINANCIALEDGE<sup>7</sup>

## Converting FX Option Risk Metrics



Delta



Gamma



Vega and Theta



### Vega and Theta

Can be multiplied by the **notional (in base currency)**  
to get their amounts in the **quoted currency**

	<u>1.10 call</u>
<b>Notional (EUR)</b>	10,000,000
<b>Vega</b>	0.00301
<b>Vega (USD)</b>	30,100
<b>Theta</b>	-0.00006
<b>Theta (USD)</b>	-622

#### Vega $\nu$

Naturally quoted **per 1%**  
absolute move in vol

May **reduce increment** to see  
our **risk** to a more  
**realistic move** in vol

**Not** all vols  
move by the **same**  
**fixed increment**



vΘ

## Vega and Theta

Can be multiplied by the **notional (in base currency)**  
to get their amounts in the **quoted currency**

	<u>1.10 call</u>
<b>Notional (EUR)</b>	10,000,000
<b>Vega</b>	0.00301
<b>Vega (USD)</b>	30,100
<b>Theta</b>	-0.00006
<b>Theta (USD)</b>	-622

**Vega** v

10%

11%

15%

16.5%

Possibly gives a more **useful**  
**aggregate** Vega number

FINANCIALEDGE<sup>7</sup>

vΘ

## Vega and Theta

Can be multiplied by the **notional (in base currency)**  
to get their amounts in the **quoted currency**

	<u>1.10 call</u>
<b>Notional (EUR)</b>	10,000,000
<b>Vega</b>	0.00301
<b>Vega (USD)</b>	30,100
<b>Theta</b>	-0.00006
<b>Theta (USD)</b>	-622

**Theta** Θ

Naturally quoted **per day**

Can be changed to **per hour**  
for very short-dated options

FINANCIALEDGE<sup>7</sup>

FINANCIALEDGE<sup>7</sup>  
[www.FE.training](http://www.FE.training)

Please do not redistribute these materials without the  
express permission of Financial Edge Training.