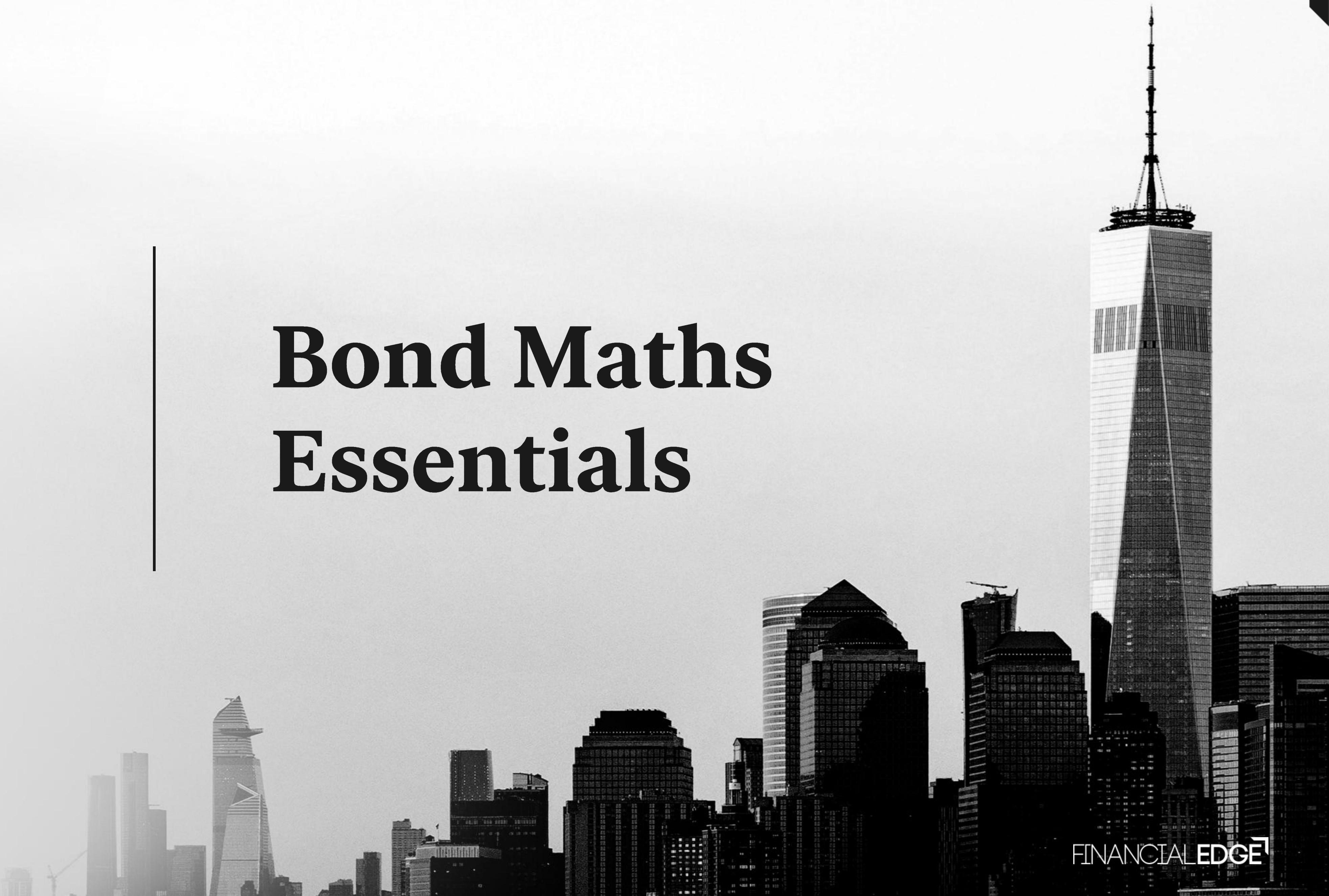
Bond Maths Essentials



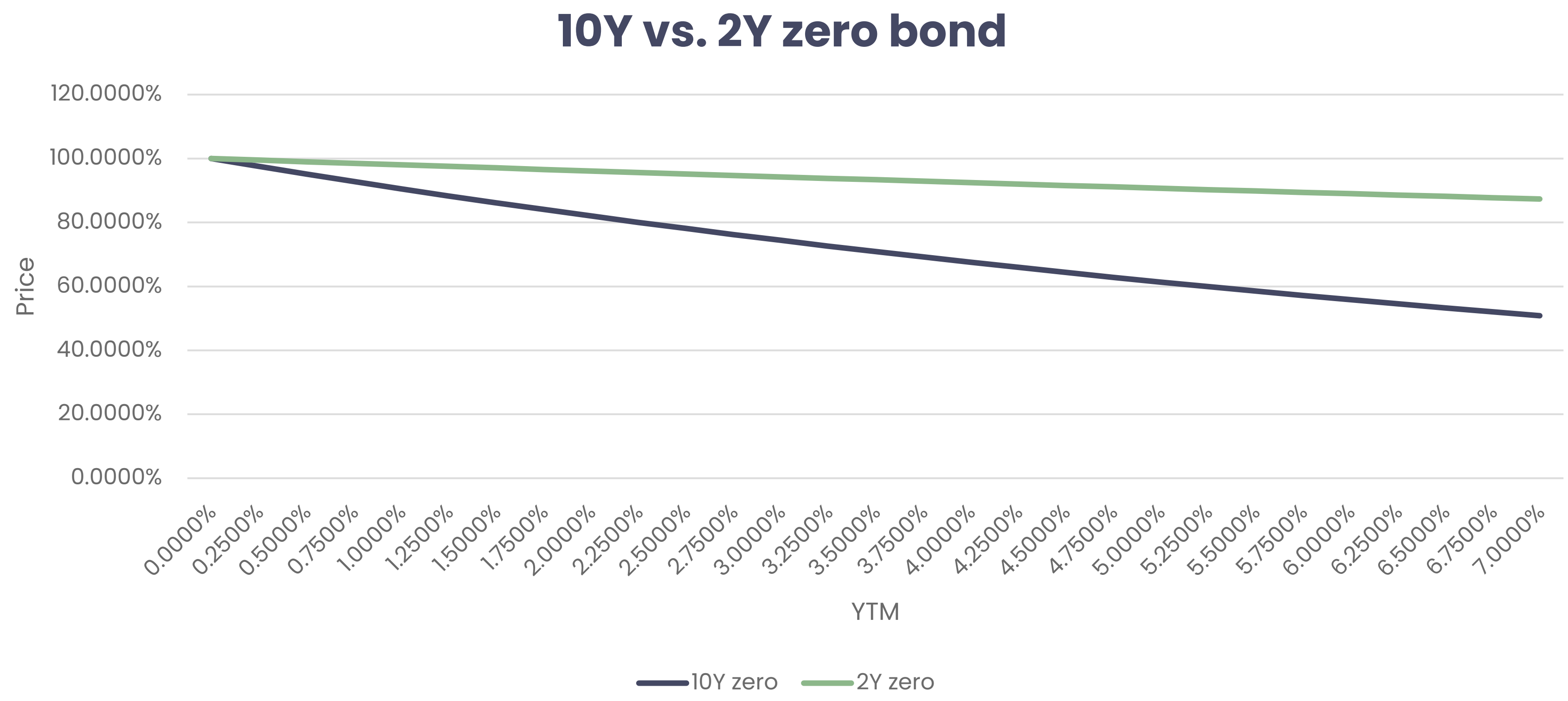
Interest Rate Risk of Fixed Coupon Bonds

Bond prices and **yields** are inversely related

Interest rate risk often defined as the **risk of a fall in bond prices** as yields rise

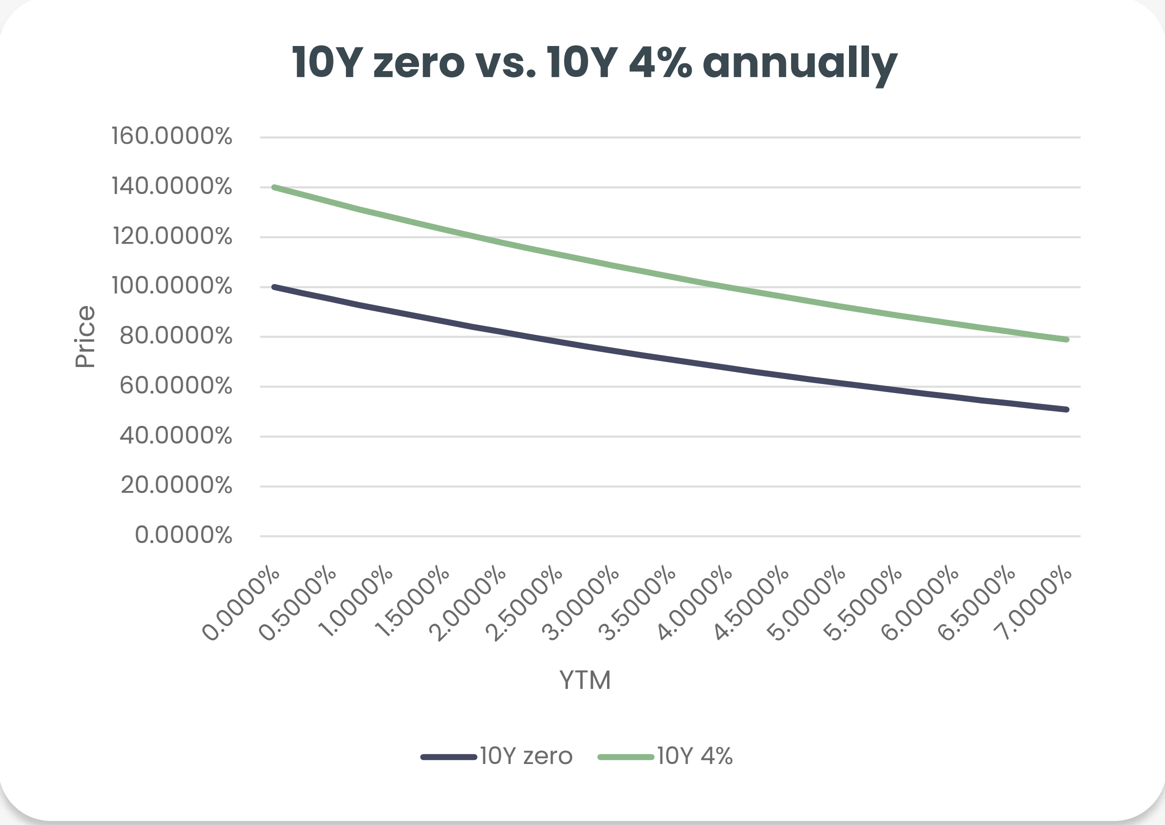
Interest rate sensitivity (or duration) describes the magnitude of the price move

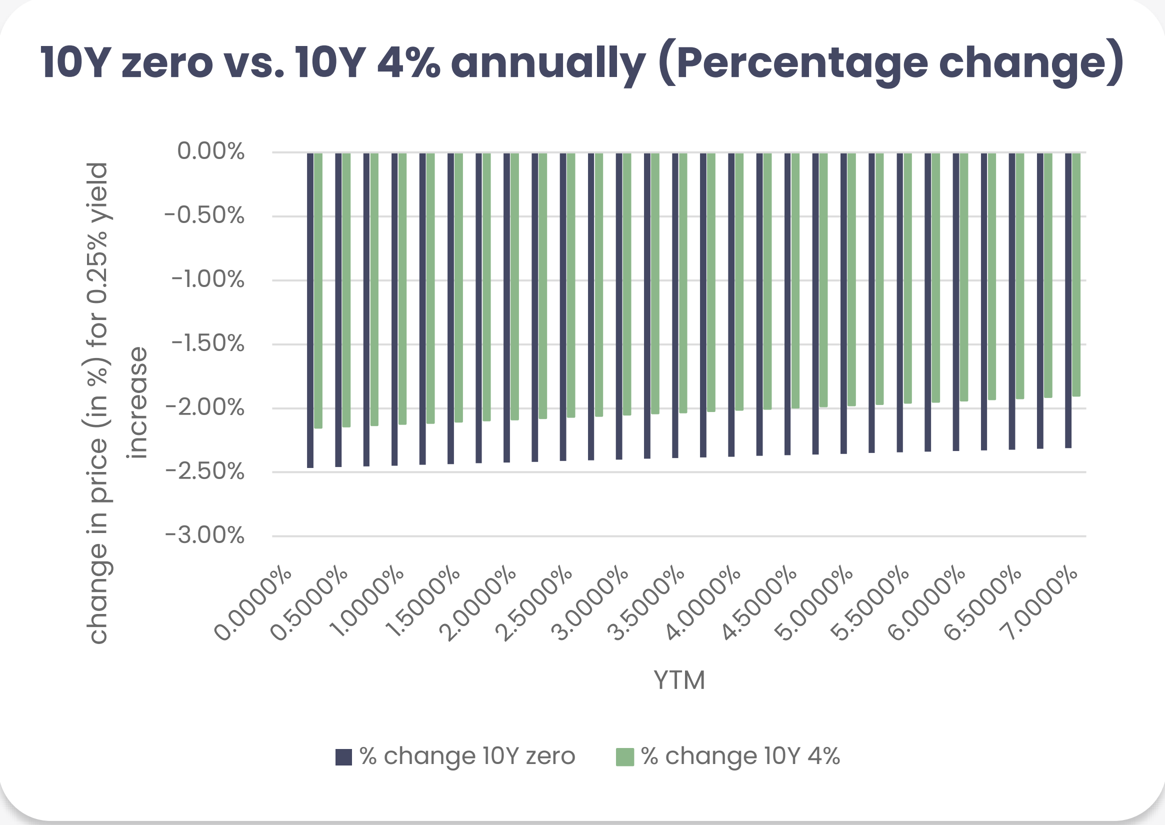
Key Drivers of IR Sensitivity - Maturity



The longer the time to maturity, the higher the interest rate sensitivity of a fixed coupon bond

Key Drivers of IR Sensitivity - Coupon





The higher the coupon, the lower the interest rate sensitivity of a fixed coupon bond

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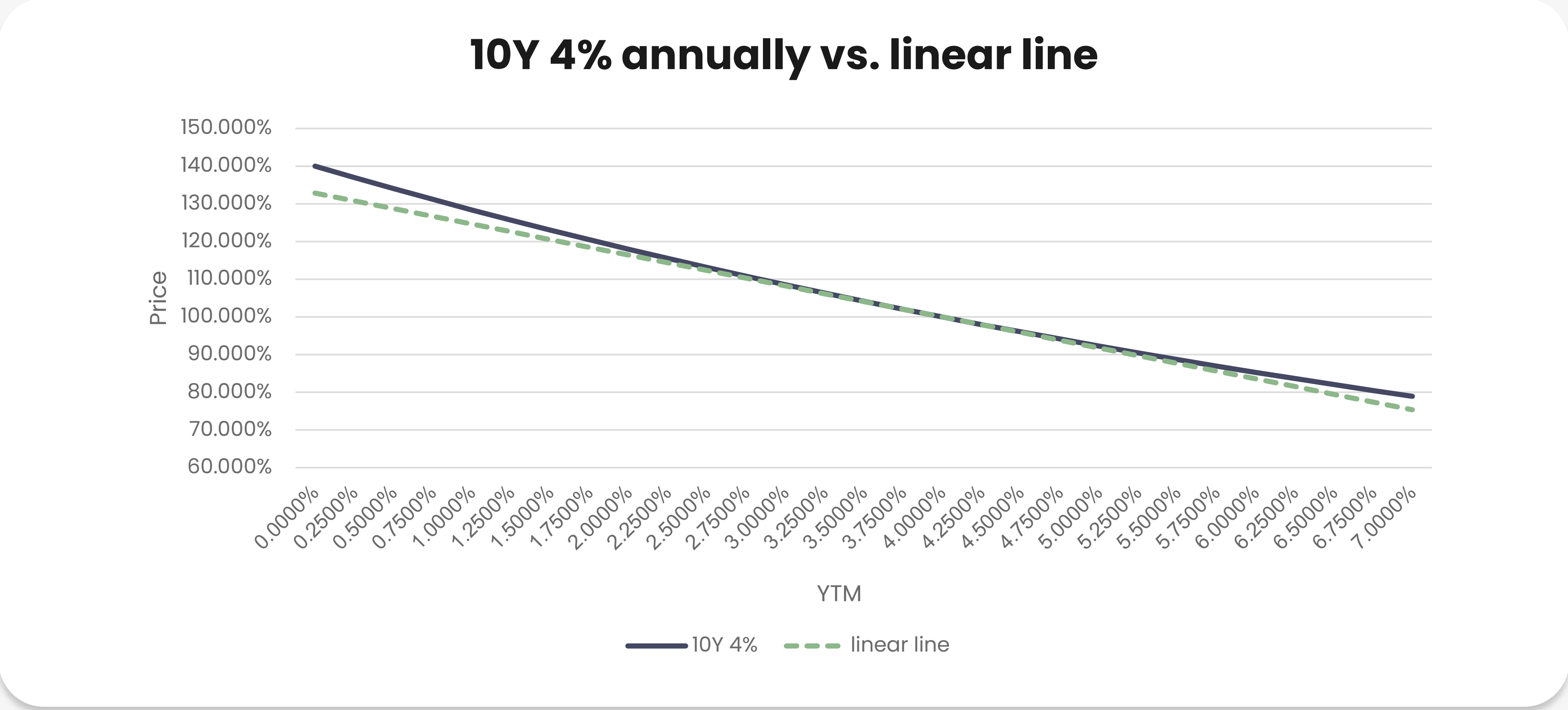
Key Drivers of IR Sensitivity – Yield Level



As yield levels rise, the interest rate sensitivity of a bond declines and vice versa

IR Sensitivity Ratios

Sensitivity ratios have been developed to make **interest rate sensitivities** of bonds with **different maturities** and **payment patterns** directly **comparable**

Macaulay duration	Modified duration	DV01

Allow direct comparison of IR sensitivity of different bonds (Macaulay duration, Modified duration, DV01)

Allow P&L impact estimates of IR scenarios (Modified duration, DV01)

It's important to note that sensitivity ratios for fixed coupon bonds are not static:

- They change as yields change
- As the bond nears its maturity, they progressively converge to zero

Macaulay Duration

The first duration measure

Weighted-average time to maturity of the bond's cash flows

Measured in years

Present value of cash flow

Time until cash flow is received

$$D_{Mac} = \frac{\sum_{t=1}^{n} CF_t * (1+YTM)^{-t} * t}{\sum_{t=1}^{n} CF_t * (1+YTM)^{-t}} = \frac{\sum_{t=1}^{n} CF_t * (1+YTM)^{-t} * t}{P_d}$$

The higher the Macaulay Duration of a bond, the higher the **bond's sensitivity**

For coupon bearing bonds, duration is always **less than their time to maturity.**

For zero-coupon bonds it is **the same as time to maturity**

Please note: The formulas shown assume annual coupon payments

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Macaulay Duration Example

5Y bond

Annual coupon: 2.4%

YTM: 2.43%

Year (t)	Cash flow	PV	PV*t
1	2.4%	2.3431%	2.3431%
2	2.4%	2.2875%	4.5750%
3	2.4%	2.2332%	6.6996%
4	2.4%	2.1802%	8.7209%
5	102.4%	90.8164%	454.0818%
	Total:	99.8603%	476.4204%

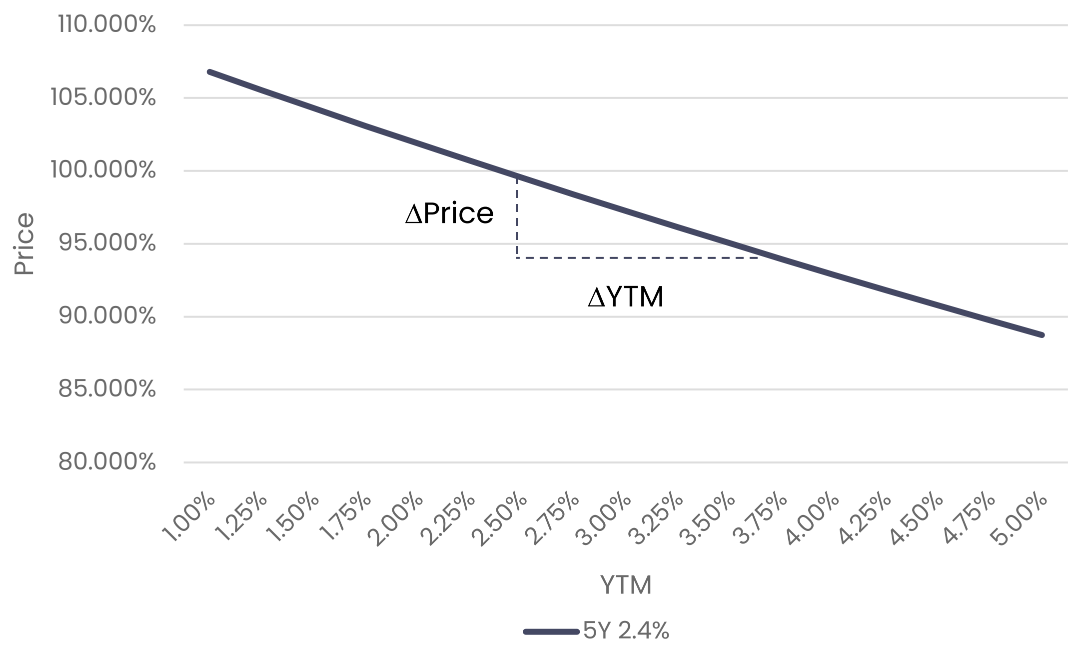
D_{Mac}

= 476.4204%/99.8603%

= 4.7709 years

Modified Duration

Approximates the percentage change of the bond price due to a change in YTM



$$D_{Mod} = \frac{D_{Mac}}{(1+YTM)}$$

To approximate the absolute change in bond price, the modified **duration must be multiplied** with the **current bond price**

Modified Duration Example

5Y bond

Annual coupon: 2.4%

YTM: 2.43%

D_{Mac} = 4.77

D_{Mod} = 4.77/1.0243 = 4.6577

Predicted relative price change for 1% yield increase:

$\%\Delta Pd = -D_{Mod} * \Delta YTM$ = -4.6577 * 1% = -4.6577%

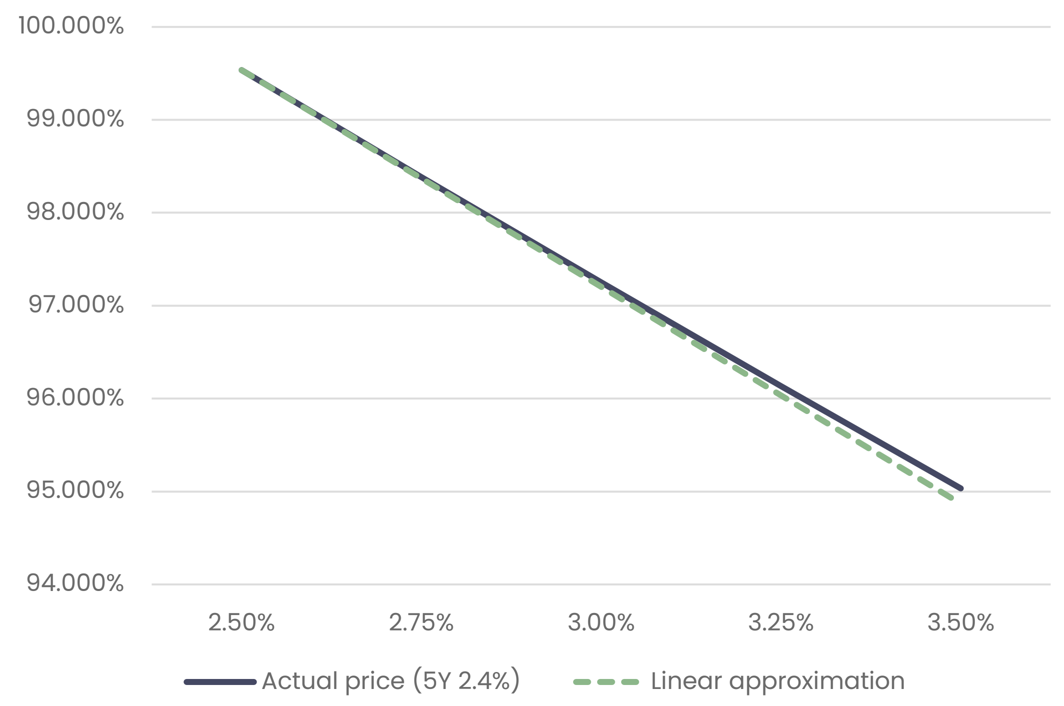
Predicted absolute price change for 1% yield increase:

$\Delta P = -D_{Mod} * \Delta YTM * P_d$ = -4.6577 * 1% * 99.8603% = -4.6512%

Predicted bond price for a YTM of 3.43%:

99.8603% - 4.6512% = 95.2091%

Convexity



Modified duration approximates the bond price assuming a linear relationship

However, the relationship between bond price and yield is convex

For small changes in YTM the approximated values will be close to the real values

For larger changes, there will be an increasing prediction error (price increases are underestimated, price decreases are overestimated)

DV01

Dollar value of 1 basis point

Shows actual P&L impact in $ terms for a 1 basis point change in yields

For single bonds DV01 can be calculated based on D_{Mod}.

$$DV01 = -D_{Mod} * 0.01\% * P_d * face\ value$$

DV01 considers both: the sensitivity of a bond and the position size, i.e. face value

DV01 Example

5Y bond

Annual coupon: 2.4%

YTM: 2.43%

$D_{Mod} = 4.6577$

$P_d = 99.8603\%$

Face value: 100 million

Predicted P&L impact of a 1 basis point increase in YTM:

DV01 = -4.6577 * 0.01% * 99.8603% * 100,000,000 = -46,511.80

Interest Rate Sensitivity for Bond Portfolios

Portfolio Duration

Often calculated as the weighted average of the individual bond durations:

$$D_{MacPort} = portfolio\ weight\ bond\ A * D_{Mac}\ of\ bond\ A + portfolio\ weight\ of\ bond\ B * D_{Mac}\ of\ bond\ B + \ldots.$$

$$D_{ModPort} = portfolio\ weight\ bond\ A * D_{Mod}\ of\ bond\ A + portfolio\ weight\ of\ bond\ B * D_{Mod}\ of\ bond\ B + \ldots.$$

$$DV01_{Port} = DV01\ \ of\ bond\ A + DV01\ of\ bond\ B + \ldots.$$

Interest Rate Sensitivity for Bond Portfolios

Interest rate delta ladder

Often calculated using step-wise yield curve shifting:

1. Define key rates
2. Calculate current portfolio value with actual yield curve
3. Shift 1st key rate by 1 basis point
4. Recalculate portfolio value
5. Change in portfolio value is 1st key rate IR delta
6. Bring 1st key rate back to starting point and repeat for all other key rates

Interest Rate Risk of FRNs

Consider the following bonds:

- A — A 5Y zero bond
- B — A 10Y coupon bond with a 4.50% coupon (paid semi-annually)
- C — A 30Y SOFR linked FRN

Which of these bonds would you expect to have the lowest interest rate sensitivity?

Interest Rate Risk of FRNs

The coupons of floating rate notes are reset periodically based **on prevailing interest rates**

The coupon reset frequency depends on the underlying reference rate

For example: daily reset in case the reference rate is an RFR like SOFR, or every 3 or 6 months if the reference rate is an IBOR rate

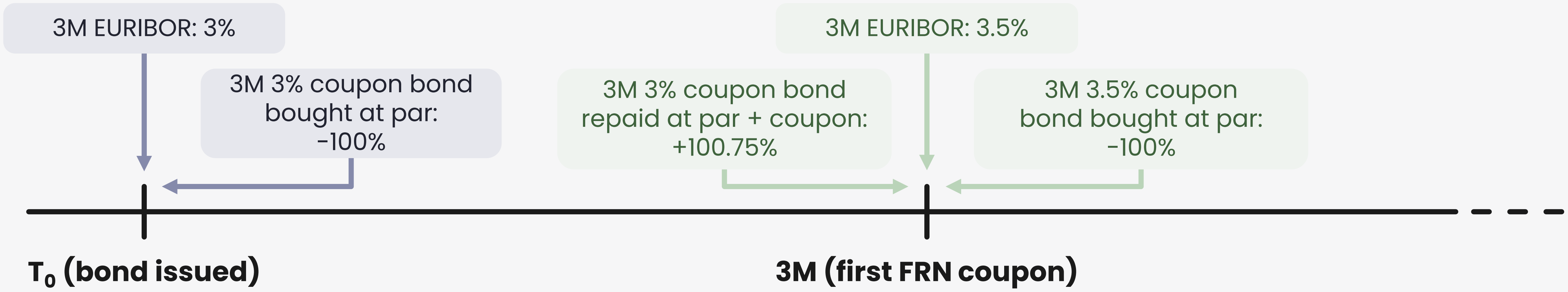
Leaving credit risk and margins(1) aside, we can conceptualize floating rate notes as a series of short-term fixed bonds, each with a duration that extends only to the next interest rate reset date

The interest rate duration of a floating rate note will therefore be equal to the time to the next coupon reset

(1) Margin (or spread) is the additional interest above the reference rate that the floating rate note will pay. The spread itself is fixed for the entire time to maturity of the floating rate note

Interest Rate Risk of FRNs

Replication of 2Y floating rate note linked to 3M EURIBOR:



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